VASAVI COLLEGE OF EDUCATION

MADAGADIPET, PUDUCHERRY-605107

STUDY MATERIAL FOR

B.ED – FIRST YEAR

PEDAGOGY OF MATHEMATICS – PART1

FULL PAPER

Compiled by

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Course - 7(vii) Pedagogy of Mathematics – Part 1_Syllabus

Unit 1: Nature and Significance of Mathematics

Meaning and Characteristics of Mathematics – Nature of Mathematics: Precision, Logical Structure, Abstractness, Symbolism– Need and Significance of Learning Mathematics. Scope of Mathematics- Mathematics in day today activities in our life, various fields, disciplines and subjects.

Unit 2: Great mathematicians and contributions

History of Mathematics –Vedic Mathematics- Contribution of eminent mathematicians for the development of Mathematics –Aryabhatta, Brahmagupta, Baskara, Ramanujam, Euler, Euclid, Pythagoras, Rene Descartes, Gauss.

Unit 3: Aims, Goals and Objectives of Learning Mathematics

Aims: Practical, Disciplinary, Cultural, Vocational, Social and Aesthetic - Taxonomy of Educational of objectives: cognitive, affective and psychomotor domains for teaching Mathematics - Revised Bloom's Taxonomy of Educational of objectives with specifications - General Instructional objectives: Knowledge, Understanding, Application, Skill, Interests, Attitude, Appreciation and Personality traits - Writing General Instructional objectives, specific learning outcomes and teaching points of various content areas in Mathematics.

Unit 4: Lesson plan preparation

Lesson planning – Meaning, Purpose, Components and Characteristics - types - needs – aspects of a good lesson plan –different models/approaches for writing lesson plan – Unit Plan, Year Plan.

Unit5: Methods and Techniques of Teaching for Facilitating Learning Mathematics

Methods of Teaching: Inductive, Deductive, Analytic, Synthetic, Lecture-cum-Demonstration, Heuristic, Laboratory, Problem solving, Project. Techniques: Oral work, Drill, Review and Assignment.

Unit 6: Models of Teaching Mathematics

Five E-Model- Engage, Explore, Enforce, Expand and Evaluate. – Suchman's Enquiry Model and Bruner's Concept Attainment Model – Individualized Instruction – Programmed Instruction – Meaning and concept – Types – Linear, Branching, Mathetics.

Unit 7: Learning Resources and diverse classroom

Mathematics textbook – workbook – Mathematics library – Mathematics laboratory – Mathematics club– Mathematics exhibition and fair – using community resources for Mathematics learning – pooling of learning resource in school complex / block / district level.

Unit 8: Professional development of Mathematics teacher

Pre–requisite qualifications – professional training: selective academic training, in-service training, professional activities, school activities, mathematical organization – administrative duties – community activities–qualities of Mathematics teacher–ethics of teacher – social and environmental responsibilities of the Mathematics teacher – problems faced by the Mathematics teachers.-Appropriate use of ICT for Teacher Professional Development (TPD) – Linkage – contribution to corporate life and to society, Journals and other resource materials in Mathematics Education – Participation in conferences/Seminars/Workshops - Qualities of a Mathematics Teacher.

Unit 9: Teaching skills

Teaching Skills – meaning, analayitical approach to understand teaching learning process in mathematics in terms of teaching skills – relevant teaching skill in teaching of mathematics – core teaching skills, meaning, components, observation procedure, writing lesson plan, for the following core teaching skills – Writing instructional objectives – Introducing a lesson, Fluency in Questioning , Probing Questioning, Explaining, Illustrating with Examples, Stimulus Variation, Reinforcement, Using Blackboard and Closure – micro teaching as a technique for acquiring teaching skills - Integration of Teaching Skills – Meaning, Need and Strategies – Vicarious integration and summation.

Unit 10: Research in Mathematics teaching

Research in the field of mathematics and mathematics teaching – status of achievement in mathematics at elementary and secondary schools – areas of difficulties – phobia for and attitude toward mathematics learning – factors related to mathematics learning – Action research –implication of research findings

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Meaning of Mathematics

Mathematics is a very important subject. Therefore, before imparting and transmitting its knowledge, it is necessary to understand, that, **'what is Mathematics?' Why its knowledge is given?** and **'What is its nature?** No one definition of mathematics is universally accepted. Generally, there are many definitions of Mathematics for example, some define mathematics as a science of calculation, some as a science of space and numbers and some as a science of measurement, magnitude and direction. Infact, the meaning of the word mathematics is—**'The science in which calculations are prime.'** In this way on the basis of these assumptions of mathematics, we can say that mathematics is the science of numbers, word , sign, etc. with which we can know about magnitude, direction & space. It is also highlighted in National Policy on Education (1986), as follows—

"Mathematics is should be visualised as the vehicle to train a child to think, reason, analyse, articulate logically. Apart from being a specific subject it should be treated as a concomitant to any subject involving analysis and meaning."

Mathematics has originated from Numbers and Number System is a special field of it, by which other branches of Mathematics are developed.

Definitions

In Hindi, Mathematics is known as 'GANITA' meaning there by—'The science of Calculations'. The term Mathematics can be defined in numerous ways to quote oxford dictionary—**"Mathematics is the science of measurement, quantity and magnitude."** Some definitions of mathematics are as follows—

1. **Marshal H. Stone**—According to Stone, "Mathematics is the study of abstract system built of abstract elements. These elements are not described in concrete fashion."

2. **Bertrand Russell**—According to him, "Mathematics may be defined as the subject in which we never know what we are talking about nor whether what we are saying true."

3. **Benjamin Peirce**—He emphasised that, "Mathematics is the science that draws necessary conclusions."

4. **Prof. Voss**—According to Voss, "Our entire civilization depending on the intellectual penetration and utilization of nature has its real foundation in the mathematical sciences.

5. **According to Galileo**—"Mathematics is the language in which God has written the universe."

6. **According to Locke**—"Mathematics is a way to settle in the mind of children a habit of reasoning."

On the basis of above definitions, we can say or conclude that—

1. Mathematics is the science of space and number.
2. Mathematics is the science of calculations.
3. Mathematics is the science of measurement, quantity and magnitude.
4. Mathematics is a systematised, organised and exact branch of science.
5. It deals with quantitative facts and relationships.
6. It is the abstract form of science.
7. It is a science of logical reasoning.
8. It settles in the mind a habit of reasoning.
9. It is an inductive and experimental science.
10. Mathematics is the science which draws necessary conclusions.

Nature of Mathematics

What is Mathematics and how does it grow are the basic questions which all the students of Mathematics must understand. In school, those subjects which are included in the curriculum must have certain aims and objectives on the basis of which its nature is decided. Mathematics holds a strong and unbreakable position as compared to other school subjects. With this reason, mathematics is more stable and important than other school subjects. The way in which the structure of a subject becomes weak, its truthfulness, reliability and prediction also decreases in the same manner. On the basis of this specific structure, the nature of each subject is determined and placed in the school curriculum.

It is not necessary that all subjects have same nature. Mathematics has its unique nature thus on the basis of which we can compare it with other subjects. The basis of comparison of two or more subjects is their nature. We can understand the nature of Mathematics on the basis of following features—

1. Mathematics is a science of space, numbers, magnitude and measurement.
2. Mathematics has its own language. Language consists mathematical terms, mathematical concepts, formulae, theories, principles and signs, etc.
3. Mathematics is a systematised, organised and exact branch of science.
4. Mathematics involves conversion of abstract concepts in to concrete form.
5. Mathematics is the science of logical reasoning.

1.2 Characteristics of Mathematics

Mathematics has certain unique features which one could hardly find in other disciplines. The following are the important characteristics of mathematics.

- Logical Sequence
- Structure
- Precision
- Abstractness
- Symbolism

1.2.1 Logical Sequence

Mathematics possesses the characteristic namely logical sequence. The study of mathematics begins with a few well-known uncomplicated definitions and postulates, and proceeds, step by step, to quite elaborate steps. It would be difficult to find a subject, in which a better gradation is possible, in which work can be adapted to the needs of the pupil at each stage, than in mathematics. Mathematics learning always proceeds from simple to complex and from concrete to abstract.

It is a subject in which the dependence on earlier knowledge is particularly great. Algebra depends on arithmetic, the calculus depends on algebra, dynamics depends on the calculus, and analytical geometry depends on algebra and elementary geometry and so on. Even within these topics, the same dependence is found.

1.2.2 Structure in Mathematics

In English dictionaries we may find that, generally speaking, a structure denotes 'the formation, arrangement, and articulation of parts in anything built up by nature or art' It seems reasonable to assume then that a mathematical structure should be some sort of arrangement, formation, or result of putting together of parts.

For example, we take as the fundamental building units of a structure the members a, b, c,... of a non-empty set 'S'. We hold together these building units by using one or more *operations*.

The familiar operations of addition denoted by +, and multiplication denoted by x, of natural numbers are operations on set N of natural numbers. Subtraction is not an operation on the set of natural numbers since the difference of two natural numbers may not be a natural number (Example:

$3 \in N, 6 \in N\ 3-6=-3 \notin N$). But subtraction is an operation on the set I of all integers.

If 'S' is a non-empty set on which one or more operations have been uniquely defined with respect to an equivalence relation, then set S together with the operation or operations is called a *mathematical system*. We will denote such a system consisting of a set S and an operation O by <S; O>. A system consisting of a set S and two operations O_1, and O_2 will be denoted by <S; O_1, O_2>. A mathematical *structure* is a mathematical system with one or more explicitly recognized (mathematical) properties. We may create a structure from a mathematical system by making specific recognition of one or more of the commutative, associative or distribution properties that the system may have.

A structure which consists of a mathematical system <S; O> with one operation, in which the operation O is associative, is called a semi-group. To illustrate, the set of positive integers under addition is a semi group, as is also the set of positive integers under multiplication.

A structure which consists of a mathematical system < S;O_1,O_2 > with two operations in which each of the operations O_1 and O_2 is commutative and associative, and in which one of the operations is distributive with respect to the other is called a *number system*. A *number* is a member of the set S in a number system. Among the familiar number systems are the following three: the natural number system <N; +,• >, the non negative integral number systems <No; +,•> (here No denotes the set of non-negative integers) and the integral number system <I; +,•>.

A semi-group with an identity number is called a monoid. The semi group of positive integers under multiplication is a monoid, for 1 is an identity number of N under multiplication. On the contrary, the semi-group of positive integers under addition is not a monoid, for the set N has no identity number under addition.

In addition to the structure mentioned above (number systems, semi-groups, monoids) there are many others such as fields, groups, vector spaces, rings etc.

With one or more basic structures at hand, one may construct other structures. Since plane analytic geometry is the study of subset of the cartesian set Re X Re where Re is the set of real numbers, plane analytic geometry may be considered as a superstructure based upon the structure known as real number system. Thus mathematics has got definite logical structures. These structures ensure the beauty and order of mathematics.

The capacity to group and order the environment in a logical way is to be gradually developed in the students. The teacher should demonstrate the beauty of mathematical structures for a greater understanding of the various mathematical structures and their interrelationships. Such an approach will help the students in appreciating the order and beauty of mathematics.

1.2.3 Precision

Mathematics is known as an '*exact*' science because of its *precision*. It is perhaps the only subject which can claim certainty of results. In mathematics the results are either right or wrong, accepted or rejected. There is no midway possible between right and wrong. Mathematics can decide whether or not its conclusions are right. Mathematicians can verify the validity of the results and convince others of its validity with consistency and objectivity. This holds not only for the expert, but also for anyone who uses mathematics at any level.

Even when there is a new emphasis on approximation, mathematical results can have any degree of accuracy required. Although *precision* and *accuracy* are distinctly different as criteria for the measures of approximation, they can be most effectively discussed when contrasted with each other. The most effective measures of both precision and accuracy are in terms of the errors (positive or negative) involved. The *precision* of a measure or a computation is evaluated in terms of the *apparent error*. The *accuracy* of a measure or a computation is evaluated in terms of the *relative error* or percent of error made.

It is the teacher's job to help the students in taking decisions regarding the degree of accuracy which are most appropriate for a measurement or calculation. This is possible by encouraging the students to observe critically, perceive relationships, analyse the data and arrive at precise conclusions/ inferences to the level of accuracy required.

1.2.4 Abstractness

Mathematics is abstract in the sense that mathematics does not deal with actual objects in much the same way as Physics. But, in fact, mathematical questions, as a rule, cannot be settled by direct appeal to experiment. For example, Euclid's lines are supposed to have no width, and his points no size. No such objects can be found in the physical world. Euclid's geometry describes an imaginary world which resembles the actual world sufficiently for it is a useful study for surveyors, carpenters and engineers.

Infinity is something that we can never experience and yet it is a central concept of mathematics. Our whole thinking is based on the assumption that there are infinitely many numbers, so that counting need never stop; that there are infinitely many fractions between 0 and 1, that there are infinitely many points on the circumference of a circle etc. We have no way of knowing and justifying that it is so because we cannot observe and count all these. Infinity, then, is not a concept corresponding to any object that we have seen or likely to see. It is an abstract concept.

Again someone whose thinking was essentially physical might refuse to believe in negative numbers on the ground that you cannot have a quantity less than nothing. Still more, such a person would refuse to believe in the square root of minus one.

Children form concepts out of experiences and lead to certain structures. Furthermore , the same structures should, if possible, be met in different situations. Children eventually learn that there is something same about four beads, four chairs, four chocolates, four friends and eventually they extract the notion four. This process of abstraction comes from their experiences of dealing with discrete objects.

However, all mathematical concepts cannot be learned through experiences with concrete objects. Some concepts can be learned only through their definitions and they may not have concrete counterparts to be abstracted from. Most of the mathematical concepts are such concepts without concretisation and hence they are abstract. The concept of a prime number, concept of probability, the concept of a function, the concept of a limit, the concept of a continuous function, to list a few are all abstract in the sense that they can be learned only through their definitions and it is not possible to provide concrete objects that correspond to such concepts. Even those concepts which one argues to be concrete are also abstract. For example, one could argue that concepts such as a point, a line, a ray, a diagonal, a circle (to list a few) can be learned through observation of concrete instances and therefore, they are concrete. But a line drawn on a board, or a dot (point), a figure of a circle are all mere representations of the concepts and they are not objects themselves. Moreover, a student learning a concept by mere observation of such instances can form wrong concepts. For example, a student can identify a figure which is not quite a circle as a circle. If the child has learned the concept by its definition as a 'closed curve on which every point is equidistant from a fixed point called centre' the child looks for the correct conditions for a curve to be a circle. Wherever possible, it is always advisable to provide the suitable concrete experiences which will lead to a generalisation forming an abstract concept.

1.2.5 Symbolism

The language for communication of mathematical ideas is largely in terms of symbols and words which everybody cannot understand. There is no popular terminology for talking about mathematics. For example, the distinction between a number and a numeral could head the list. A number is a property of a set; that property tells how many elements are there in a set. A numeral is a name or a symbol used to represent a number. Essentially, to distinguish between a number and a numeral is to distinguish between a thing and the name of a thing. If the things considered are physical entities, there seem to be little difficulty in making distinctions. But if things are abstract entities such as those we deal with in mathematics, it becomes considerably more difficult to make the distinction between the name of the thing and its referent, the thing itself. Since numbers are abstractions and cannot be perceived by any of the five senses, they are often confused with their names. A teacher ought to be very careful to use correct terms, since this helps children to learn and think better. It is important that a student understands the distinction between a number and a numeral so that he may realise the differences between actually operating with numbers and merely manipulating symbols representing those numbers. This is only one item in regard to precision of language. There are many others, such as distinguishing between the line and picture of a line, a point and the dot used to represent a point, to list a few.

Without language, we cannot talk about any thing. Mathematical talk consists of making use of mathematics symbolism. Understanding mathematics is realising what symbolism corresponds to the structure that has been abstracted. It is not enough for children to understand mathematics; it is necessary for them to speak mathematics; in other words to handle symbols. This corresponds to speaking a language as opposed to understanding a language. The process of speaking of the mathematical language runs as follows: an *abstraction process,* followed by a *symbolisation process,* followed again by the *learning of the use of the symbols.*

There are great dangers in a purely symbolic treatment. By purely symbolic, it is meant that no reference is being made to the entities symbolised while the symbols are being used. The danger is that the symbols have not acquired the same firm meanings as other word symbols currently used in the language used by the children. The amount of experience which lies behind the vocabulary used by children in their own languages is enormous. It is not possible to hope to put the same amount of experience behind mathematical symbols. Symbolic or even verbal statements of a concept are meaningless

unless the symbolism is related to something real and concrete. Thus, if children are presented with symbols before they have abstracted the concepts that the symbols represent – the only way they can deal with them is associatively. They treat them as nonsense and learn by rote.

In arithmetic and algebra students' deal not with facts, but with symbols. The child who is poor in mathematics is unable to see what concepts the symbols stand for, what the concepts themselves are abstracted from, and hence what the symbols communicate.

The symbols of mathematics constitute a language which is gradually developed by and for the pupil. This language must be acquired just like any other language and there is need for translating this language into one's own mother tongue. Long periods of training with patience and endurance is needed to make the students feel at home with this language. The training that mathematics provides in the use of symbols is an excellent preparation for other sciences.

The use of symbols makes the mathematical language more elegant and precise than any other language. For example, the commutative law of addition and multiplication in real number system can be stated in the verbal form as: 'the addition and multiplication of two real numbers is independent of the order in which they are combined'.

This can be stated in a concise form as: $a + b = b + a$, and $a * b = b * a$, where a & b are elements of R . Almost all mathematical statements, relations and operations are expressed using mathematical symbols such as $+, -, \times, \%, <, >, \equiv, \leq, \sqrt{}, \Sigma, \varepsilon$ and so on. It is highly impossible to prepare a comprehensive list of all the mathematical symbols. Anyone, who wants to read and communicate effectively in the mathematical language, has to be well versed in the mathematical symbols and their definite uses.

CORRELATION IN MATHEMATICS

The social sciences mathematically developed are to be controlling factors in civilization. —*W. F. White*

According to webester's new dictionary of English language correlation means "The act of correlating[2] or "the state of being correlated" or "mutual relation of two or more things, or parts." Thus the word 'correlation ' means the reciprocal relationship between various subjects of the curriculum. In other words whenever two variables 'a' and 'b' are so related that a change in one is accompanied by change in the other in such a way that an increase in the one is accompanied by an increase or decrease in the other, then variables are said to be correlated. If two variables vary in the same directions, the correlation is said to be positive, on the other hand if two variables vary in opposite directions, the correlation is said to be negative.

In isolation the child cannot learn the various subjects of curriculum because the child is interested to learn the things which are related to his experiences and previous knowledge. According to the principles of correlation, knowledge cannot be acquired in piece meals. Therefore, the knowledge of various study subjects must be integrated and provided to the child in the form of whole or unit

In such a way knowledge of mathematics may be more interesting and realistic. Hence, the problems of mathematics must be related with day to day experiences of the children.

Definitions

According to **Ferguson**—"Correlation is concerned with describing the degree of relation between variables."

According to **Lathrop**— "Correlation indicates a joint-relationship between two variables."

On the basis of above definitions it is clear that correlation is the reciprocals relationship between various study subjects of the school curriculum. Therefore, the knowledge of various study subjects must be integrated and related with the daily life of the child.

Types of Correlation

There are two types of correlation is mathematics—

(i) Incidental correlation
(ii) Systematic correlation

Incidental Correlation

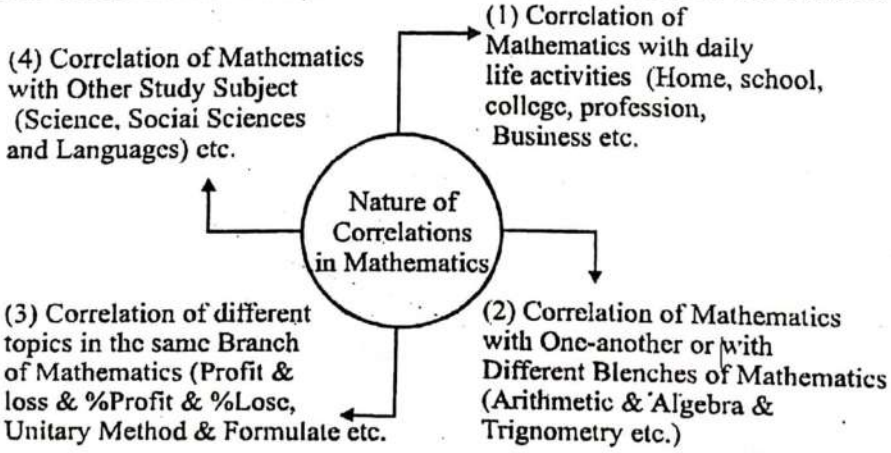
There teacher plays a prominent role. In incidental correlation, the correlation of two or different subjects is not predecided. Thus to establish this type of correlation, a teacher must have versatilie knowledge of basic elements of different subjects. Without having sufficient knowledge of different subjects a teacher cannot establish incidental correlation in mathematics. For example—If a mathematics teacher has versatile knowledge of Physics, Chemistry, Zoology etc., he can establish incidental correlation easily. Hence, the teacher of mathematics should have versatile knowledge of other study subjects also.

Systematic Correlation

To establish systematic correlation is the responsibility of curriculum constructors or framers the content of different subjects is arranged in a particular sequence or order. Here, attention is paid towards the application of the facts, principles and laws of mathematics or one subject to other subject areas. Thus the knowledge becomes more meaningful, interesting, realistic and natural.

Nature of Correlation in Mathematics

The correlation in mathematics is extended in four directions-viz—



1. Correlation of mathematics with daily life activities (Home, school, college, profession, business etc.)

2. Correlation of mathematics with one-another or with different branches of mathematics (Arithmetic & algebra, algebra & trigonometry etc.)

3. Correlation of different topics in the same branch of mathematics (Profit & loss & % profit & loss, unitary method & formulae etc.)

4. Correlation of mathematics with other study subject (Sciences, Social sciences and languages etc.)

Correlation of Mathematics with Daily Life Activities

It is true that in each and every sphere of our daily life mathematics provides us its help and patronage without which we can not do anything. The knowledge of mathematics is very versatile and required in all the day to day life activities beginning from awakening to sleep in night. Thus we can say that in our daily life mathematics is as mixed as the oxygen in the air. Every dealing of our life begins and ends with the mathematical thinking and reasoning. For example; daily house hold problems, food, clothing, idea of quantity and quality, daily account of income and expenditure, allocation of funds etc. Such type of day to day problems cannot be solved without the knowledge of mathematics. Young J.W.A. has remarked that "whenever we turn in these days of iron, steam and electricity we find that mathematics has been the pioneer were its backbone removed, our material civilization would inevitably collapse." Greate mathematician Hubsch has also empharised that, "Mathematics is like a Wheatstone and by its study one learns to think distinctly, consecutively and carefully."

Hence without the proper knowledge of mathematics literate as well as illiterate people cannot solve their daily life problems. Thus knowledge of mathematics is required in every profession be it a cobbler, a vegetable seller a merchant, a driver, a carpenter, a cashier, a labour etc. To conclude we can say that knowledge of mathematics is closely correlated with our day to day life activities. Each and every individual needs the knowledge of mathematics at every movement.

Correlation of Mathematics with Different Branches of Mathematics

Mathematics has a well established correlation within the different branches of mathematics. It can be broadly divided into three basic structures—Algebric, Topological and Order structure. All these structures

are inter related with each other. Mathematics deeply concerned with the phenomenon of correlation between its various branches. Such as—one cannot learn the four fundamental rules of arithmetic and algebra with out having knowledge of integers, without having elementary knowledge of arithmetic, algebra and geometry, one cannot solve the problem of statistics. Similarly to learn trignometry one requires the knowledge of arithmetic, algebra and geometry; integral and differential calculus, geometry and dynamics etc., all these branches are interrelated with each other. No one can understand, in isolation of any one branch of mathematics. Thus each and every branch of mathematics is interdependent to each other. Hence, to make the knowledge of mathematics more useful and meaningful, the mathematics should be taught by establishing correlation between different branches of mathematics. Therefore, all the branches of mathematics must be integrated with one-another.

Correlation of Different Topics in the Same Branch of Mathematics

Mathematics plays a vital role in establishing relationship between the numerous topics in the same branch of mathematics. For example; Profit and loss is closely related to the banking and percentage, linear and simultaneous equations can be correlation easily.

Similarly so many examples can be given in support of this types of correlation. Thus the teacher of mathematics should try to establish correlation between the different topics of the same branch. So that pupil can learn the content in a systematic way by interrelating the various topics of the same branch of mathematics. Hence, there must be a logical sequence or continuity in the different topics of same branch. The different units must be so correlated as to bring out clearly the object of teaching the whole subject-matter.

Correlation of Mathematics with Other Study Subjects

Mathematics and Sciences—Mathematics is the pivot of all sciences. It has been very well said that mathematics is a science of all sciences and art of all arts, In this respect 'Roger Bacon' has remarked that, "Mathematics is the gateway and key of all sciences." The correlation of mathematics with different branches of science is as follows—

(a) ***Mathematics and Physics***—Mathematics and Physics are very closely related and has a very well established correlation. Each rule and principle in physics takes mathematical form and mathematics gives final shape to the rules, laws and principles of physics. For example—

- The laws of motion, friction, expansion of solids, liquid pressure are dependent on mathematics.
- The measurement unit in physics is employed as frequently as in mathematics.
- The coefficient of linear expansion of deferent metals, cubical expansion of liquids, charles' law of expansion of gases, conversion of the scales, principle of working of lenders, are based on mathematic knowledge.

(b) ***Mathematics and Chemistry***—Mathematics and chemistry are also correlated. All the chemical combinations and their equations are based on mathematics. For example—

- The formation of various compounds like H_2S. The formation of H_2S gas is possible only when two atoms of hydrogen are combined with one atom of sulphur. Without this mathematical ratio and proportion, preparation of H_2S is not possible.
- In the estimation of elements of organic compound, the use of percentage, ratio & proportion is needed.
- Molecular weight & number of organic compound, is based on mathematics.
- Balancing of chemical equations and other calculation work is also based on mathematical knowledge.

(c) ***Correlation of Mathematics with Biology***—A biologist must have the sufficient knowledge of mathematics to understand its branches like; biophysics and bio-chemestry. For example—

- The caloric and nutritive values of food articles are based on mathematics.
- The rate of respiration, transpiration, supply of water in connection to all living bodies, is related to mathematics.
- Number of bones, organs etc., it is also mathematical.
- Mathematics can acquire knowledge of heredity, growth, nutrition, maturation, etc.
- Study of living cells, composition of blood, age & category of plants, animals, is possible through mathematical knowledge.

(d) ***Mathematics and Engineering***—Mathematics is the foundation of engineering sciences. As we know that engineering deals with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc., All these processes needs knowledge of

mathematics. Hence, high level mathematical knowledge is required for study of engineering subjects. Its exaltation is impossible without mathematical knowledge.

(e) ***Mathematics and Agriculture***—The knowledge of mathematics is also very necessary in the field of agriculture. In agriculture we need mathematics knowledge in numerous way viz—

- Investment of money, energy, time and its return.
- Measurement of land, area, production per unit area, rate of seeds. labour cost etc.
- Progress of production can be judged with the help of graph and other statistical techniques.
- Estimation of average production, profit and loss etc.

Correlation of Mathematics with fine Arts & Drawing

Great mathematics Pythagoras has well remarked that, "where, there is money, there are numbers." Mathematics is considered as the science of reality and truthfulness, Keats has also said that, "Truth is beauty." In the study of mathematics the child has to draw and construct various shapes, sketches, figures, graphs etc. These all activity requires the knowledge of drawing and arts. It is evident that good drawing is needed to draw good geometrical figures . Similarly in fine arts, the mathematical knowledge is applied in many ways, e.g..

- Drawing and painting with symmetry.
- Ratio and proportion in drawing, painting, sketching etc.
- Almost all musical notes and systems work or mathematical principles.
- Exactness of a figure, shape etc. can be measured with the help of mathematics.

Here in this support we can also quote leibnitz, he remarked that 'music' is a modern hidden exercise in arithmetic of a mind unconscious of dealing with numbers. Thus, there is a close correlation between mathematics and fine arts and drawing.

Correlation of Mathematics with Economics

Mathematics laws, language, principles and methods are frequently used in describing, interpreting and explaining economic phenomena. According to **J.B. say**—"Economics is the science which treats of wealth.." Great economist Marshal has well remarked that, " The direct application of mathematical reasoning to the discovery of economic truths

has recently rendered great services in the hands of master mathematics." In economics mathematical knowledge is applied in many ways such as—

Collection analysis, calculation, interpretation and presentation of data regarding, population, investment, income, expenditure, production, banking, business, demand and supply etc.

Various mathematical methods are used to economic forecasts regarding trade cycles, trends of export and import, employment, production etc.

Almost all magazines, news papers etc., are full of figures and graphs regarding population, investment, employment, production, crops, marketing etc. All these employ various numerical and statistical procedures.

This is fact that mathematics play prominent role in economics. Hence the correlation between mathematics and economics is very vast.

Correlation of Mathematics with History

The knowledge of history helps in understanding the historical development of mathematics. History and contribution of great mathematicians in the field of mathematics. Similarly mathematical knowledge is applied in history to know the dates, time, days etc., of various historical events. Thus the correlation between history and mathematics is reciprocal.

Correlation of Mathematics with Geography

The knowledge of mathematics is also applicable to the study of geography. For example—shape and size of earth, area, distance of places, height and distance, temperature longitude, time etc. all are based on mathematical calculation. Similarly, to study the rivers, canals, mountains, population, climate, depth, drawing of graphs, and figures, moment of winds, falling of rains etc. in all these, mathematics plays a prominent role. Hence all the geological and geographical studies are not possible without the proper application of mathematics.

Correlation of Mathematics with Commerce

The knowledge of mathematics helps to understand various concepts and branches of commerce. The basis of banking and accountancy is mathematics. It is considered that without having good knowledge and practice of mathematics one cannot become a good accountant. In commerce calculation of interest etc. all are based on mathematical principles and formulae. Shares, debentures, Mutual funds,

accounting etc. are also based on mathematical calculation. Thus most of the knowledge of commerce requires the application of mathematical knowledge. Many statistical techniques are also used to solve the problems of different brances of commerce.

Correlation of Mathematics with Logic

Logic is the only branch of knowledge in which logical reasoning and laws are applied. The results can be varied with the help of logical reasoning. Logic is the scientific study of condition of accurate thinking and valid influence. In this respect Pascal, a great mathematician, has well said that, 'Logic has borrowed rules from geometry.' 'D' Alombert' also emphasized that, "Geometry is a practical logic because in it rules of reasoning are applied in the most simple and sensible manner." There fore, the aims of mathematician and logicians are the same. In mathematics the symbols can be use logically such as – $a > b$, $b < a$, $a = b$, $A \cap B$, $B \subseteq A$ etc.

Correlation of Mathematics with Languages

To study mathematics, language plays very important role because no subject can be taught without language. Various principles, laws, formulae are expressed and generalized through the language. It will be very difficult to express the knowledge of mathematics without the medium of language. To convey the correct and exact idea of mathematical facts, concept, laws etc. language is necessary. Language also helps in developing the various mental powers of the child like-memory, imagination, expression, observation etc.

Correlation of Mathematics with Psychology

Learning of mathematics from psychological point of view is also very important. While learning mathematics various psychological principles, laws, theories and concepts are applied to make the learning child centered. Some psychological principles like—learning by doing, learning through experiences, learning by problem-solving, law of readiness, law of exercise etc. all are very useful to understand mathematics. Similarly, various mathematical and statistical techniques are used to collect, to analyses and to interpret psychological data. Experimental psychology is also based on mathematical calculations and applications. In this support we can quote Herbrt, he remarked that, "It is not only possible but necessary that mathematics be applied to psychology."

Correlation of Mathematics with Recreation

The word 'Recreation' means refreshment after labour. Everyone needs recreation after the tedious work either a child or a grown up person. In other words recreation is to entertain, to gratify, to create again. Thus recreation is the act of relaxing and amusing oneself after work. The main advantage or importance of recreation is that shapes the thinking and imagination power of a person of any age group. That is why recreation is also related to mathematics and involves many problems which can be solved by the application of mathematical knowledge. As far as, the correlation of mathematics with recreation is concerned, recreation in mathematics implies different kinds of mathematical puzzles, riddles and cross word games or puzzles. These all activities are helpful in raising up the mental level of the child and plays a significant role in the development of different mental abilities of the child. While solving such type of games and puzzles the child gives stress to his mind. Thus these activities help in making his thinking and imagination powers more powerful and strong. After solving these mathematical games, puzzles and riddles, a feeling of self appreciation develops in the mind of the child. So that the child becomes more confident and the feeling of competition is also developed in his mind.

Apart from the puzzles and riddles, the children form different groups and play various games which require mathematical knowledge. By playing and performing these games, the children recall there previous knowledge and experiences, so that their mind becomes sharp and mental faculties are developed. Thus the more child indulges himself in these games and develops his mental abilities. The popular saying that, "Practice makes a man perfect" becomes true while performing such type of mathematical puzzles, riddles and games.

Hence, we can conclude that to solve the various puzzles, riddles and mathematical tricks etc., the knowledge of mathematics plays an important role. Therefore, recreation and mathematics are correlated with each-other in various aspects.

In this way correlation of mathematics with some more other study subjects may be explained. Here only few subjects have been discussed. On the basis of above discussion it can be concluded that while teaching mathematics, a mathematics teacher should establish the correlation of mathematics with other study subject. So that the children can understand the importance and utility of mathematics in his life.

Need and Significance of learning Mathematics

- Mathematics is a powerful tool for global understanding and communication that organizes our lives and prevents chaos.
- Mathematics helps us understand the world and provides an effective way of building mental discipline.
- Math encourages logical reasoning, critical thinking, creative thinking, abstract or spatial thinking, problem-solving ability, and even effective communication skills

Mathematics: Introduction

- Mathematics is a study of measurements, numbers, and space, which is one of the first sciences that human work to develop because of its great importance and benefit.
- The origin of the word "mathematics" in Greek, which means tendencies to learn, and there are many branches of mathematics in science, that are related to the numbers, including the geometric forms, algebra, and others.
- Mathematics plays a vital role in all aspects of life, whether in everyday matters such as time tracking, driving, cooking, or jobs such as accounting, finance, banking, engineering, and software. These functions require a strong mathematical background, and scientific experiments by scientists need mathematical techniques. They are a language to describe scientists' work and achievements.
- As for mathematical inventions, they are numerous throughout the ages. Some of them were tangible, such as counting and measuring devices. Some of them are not as tangible as methods of thinking and solving. The symbols that express numbers are also one of the most important mathematical inventions.
- Mathematics helps in analytical thinking. While solving math problems, data are collected, disassembled and then interconnected to solve them.
  1. Mathematics helps to develop the ability to think.
  2. It helps explain how things work.
  3. It helps to develop wisdom.
  4. It increases the speed of intuitive.
  5. It helps to make the child smarter.
  6. Money can be collected in mathematics when used as a profession.
  7. It is important in a constantly evolving world.
  8. It provides the child with an opportunity to get to the world.

Mathematics in Our Everyday Life

Mathematics is the pillar of organized life for the present day. Without numbers and mathematical evidence, we cannot resolve many issues in our daily lives. There are times, measurements, rates, wages, tenders, discounts, claims, supplies, jobs, stocks, contracts, taxes, money exchange, consumption, etc., and in the absence of these sports data, we have to face confusion and chaos.

Thus, mathematics has become the companion of man and his helper since the beginning of human existence on earth. When man first wanted to answer questions such as "How many?" he invented math. Then algebra was invented to facilitate calculations, measurements, analysis, and engineering.

The science of trigonometry emerged when humans wanted to locate high mountains and stars.Therefore, the knowledge of this article arose and developed when human felt the need and mathematics are necessary for the long planning of life and also the daily planning of any individual.

Mathematical rapprochement is necessary for any process, so if anyone wants to reach the height of his life, he should not fail to believe in the role of mathematics in his life, starting with the ordinary citizen. Every day has a daily interest in mathematics.
Mathematics is deeply related to the natural phenomenon, the way to solve many secrets of nature.

Mathematics is necessary to understand the other branches of knowledge. All depend on mathematics in one way or another. There is no science, art, or specialty except mathematics was the key to it. The discipline and mastery of any other science or art are very much related to the size of mathematics.

The Most Important Uses of Mathematics

- I think it is impossible to limit the uses of mathematics in everyday life so we will suffice with some of them:
- Can you use any entertainment game without using numbers?
- Can you practice any sport without using numbers to learn if you are a winner or a loser?
- Can you do your work without using the numbers? If you are a teacher, collect your students' marks or a doctor, estimate the amount of medicine for the patient or an engineer, estimate the amount of raw material to be added to complete the work or even a leader in a battle.
- Can you enter the store without using the numbers?
- Can prayers be organized without the use of numbers, and what is left of the time for the next prayer?
- And much more, whatever you try, you cannot get rid of the use of this important science.

The importance of mathematics is that it is a method based on research and analysis, to reach the desired results, and is used for calculation and presentation of data; not only the use of this science in a particular field but the use of all areas of life and different sciences.

The Importance of Mathematics to Individuals

Mathematics is one of the most important sciences that cannot be dispensed with, and the individual's need for mathematics is no less than the need of society. When studying an individual or a person of mathematics, this will develop his thinking and scientific tendencies.

Mathematics also works to express the most accurate and objective, and it helps us to manage time and planning things, economics and others.

The Importance of Mathematics to Society

Mathematics is an innate approach to research and analysis to reach a solution to mathematical problems.

Mathematics is a basic subject taught at all stages. It cannot be dispensed with. Some of the important roles of mathematics in society are:

- **The importance of mathematics in scientific studies:** The conduct of studies and scientific research requires many skills in sports, which help in the development of studies and progress, for example, the study of physics or chemistry depends heavily on mathematics, including mathematical skills and mathematical matters, and lies the role of mathematics in the accuracy of conducting social research through the statistics branch.
- **The importance of mathematics in the development of social values:** It helps mathematics in the development and refinement of personality, through the organization and accuracy, research and investigation and study, and helps to detect.
- **The importance of mathematics in cultural development:**
    - Like other sciences, mathematics influences civilization's development.
    - Cultural development depends on the achievements of scientists and innovators, especially athletes. No one neglects the role of Arab and Muslim scientists in the development of civilizations.
    - A great mathematician, Muhammad ibn Musa al-Khwarizmi who created the theory of algebra and laws of arithmetic that helped develop modern science and technology.

Vedic Mathematics

Vedic Mathematics is the name given to a supposedly ancient system of calculation which was "rediscovered" from the Vedas between 1911 and 1918 by Sri Bharati Krishna Tirthaji Maharaj (1884-1960). According to Tirthaji, all of Vedic mathematics is based on sixteen Sutras, or word-formulae. For **example**, "**Vertically and Crosswise**" is one of these Sutras. These formulae are intended to describe the way the mind naturally works, and are therefore supposed to be a great help in directing the student to the appropriate method of solution. None of these sutras has ever been found in Vedic literature, nor are its methods consistent with known mathematical knowledge from the Vedic era.

Perhaps the most striking feature of the Tirthaji system is its coherence. The whole system is interrelated and unified: the general multiplication method, for **example**, is easily reversed to allow one-line divisions, and the simple squaring method can be reversed to give one-line square roots. And, these are all easily understood. This unifying quality is very satisfying, it makes arithmetic easy and enjoyable, and it encourages innovation.

Difficult arithmetic problems and huge sums can often be solved immediately by Tirthaji's methods. These striking and beautiful methods are a part of a system of arithmetic which Tirthaji claims to be far more methodical than the modern system. "Vedic" Mathematics is said to manifest the coherent and unified structure of arithmetic, and its methods are complementary, direct and easy.

The simplicity of the Tirthaji system means that calculations can be carried out mentally, though the methods can also be written down. There are many advantages in using a flexible, mental system. Pupils can invent their own methods; they are not limited to one method. This leads to more creative, interested and intelligent pupils.

Interest in the Tirthaji's system is growing in education, where mathematics teachers are looking for something better and finding the Vedic system is the answer. Research is being carried out in many areas including the effects learning the Tirthaji system has on children; developing new, powerful but easy applications of these Sutras in arithmetic and algebra.

The real beauty and effectiveness of the Tirthaji system cannot be fully appreciated without practising the system. One can then see why its enthusiasts claim that it is the most refined and efficient calculating system known.

"Vedic Mathematics" refers to a technique of calculation based on a set of 16 Sutras, or aphorisms, as algorithms and their upa-sutras or corollaries derived from these Sutras. Its enthusiasts advance the claim that any mathematical problem can be solved mentally with these sutras.

Vedic Mathematics introduces the wonderful applications to Arithmetical computations, theory of numbers, compound multiplications, algebraic operations, factorisations, simple quadratic and higher order equations, simultaneous quadratic equations, partial fractions, calculus, squaring, cubing, square root, cube root and coordinate geometry etc.

Uses of Vedic Mathematics:

- It helps a person to solve mathematical problems 10-15 times faster
- It helps m Intelligent Guessing
- It reduces burden (need to learn tables up to 9 only)
- It is a magical tool to reduce scratch work and finger counting
- It increases concentration.
- It helps in reducing silly mistakes

Some Name of Sutras:

NAME / SUTRA	COROLLORY / SUB-SUTRA	MEANING
Ekadhikena Purvena	Anurupyena	By one more than the previous one
Nikhilam Navatashcaramam Dashatah	Sisyate Sesasamjnah	All from 9 and the last from 10
Urdhva-Tiryagbyham	Adyamadyenantyamantyena	Vertically and crosswise
Paravartya Yojayet	Kevalaih Saptakam Gunyat	Transpose and adjust
Shunyam Saamyasamuccaye	Vestanam	When the sum is the same that sum is zero
Anurupye Shunyamanyat	Shunya Anyat	If one is in ratio, the other is zero
Sankalana-vyavakalanabhyam	Yavadunam Tavadunikritya Varga Yojayet	By addition and by subtraction
Puranapuranabyham	Antyayordashake'pi	By the completion or non-completion
Chalana-Kalanabyham	Antyayoreva	Differences and Similarities
Yavadunam	Samuccayagunitah	Whatever the extent of its deficiency
Vyashtisamasthi	Lopanasthapanabhyam	Part and Whole
Shesanyankena Charamena	Vilokanam	The remainders by the last digit
Sopaantyadvayamantyam	Gunitasamuccayah Samuccayagunitah	The ultimate and twice the penultimate
Ekanyunena Purvena	Dhvajanka	By one less than the previous one
Gunitasamuchyah	Dwandwa Yoga	The product of the sum is equal to the sum of the product
Gunakasamuchyah	Adyam Antyam Madhyam	The factors of the sum is equal to the sum of the factors

HISTORICAL BACKGROUND OF MATHEMATICS AND MATHEMATICIANS

"India was the motherland of our race and Sanskrit the mother of Europe's languages. India was the mother of our philosophy, of much of our Mathematics, of the ideals embodies in christianity....of self-government and democracy. In many ways, Mother India is the mother of us all." ***—Will Durant***

—American Historian 1885-1981

Introduction

Mathematics represents a high level of abstraction attained by the human mind. In India, mathematics has its roots in Vedic literature which is nearly 4000 years old. Between 1000 A.D. and 1000 B.C. and 1000 A.D. Various treatics on mathematic were authored by Indian mathematicians in which were set forth for the first time, the concept of zero, the techniques of algebra and algorithm, square root and cube root.

1. As in the applied sciences like production technology, architecture and shipbuilding, Indians in ancient times also made advance in abstract science like Mathematics and Astronomy. It has now been generally accepted that the technique of algebra and the concept of zero originated in India.

2. But it would be surprising for us to know that even the rudiments of Geometry, called Rekha-Ganita in ancient India, were formulated and applied in the drafting of Mandalas for architectural purposes. They were also displayed in the geometic patterns used in many temple motifs.

3. Even the technique of calculation, called algorithm, which is today widely used in designing soft ware programs for computers was also derived from Indian mathematics.

The Concept of Zero

The concept of zero also originated in ancient India. This concept may seem to be a very ordinary one and a claim to its discovery may be viewed as queer. But if one gives a hard thought to this concept it would be seen that zero is not just a numeral. Apart from being a numeral, it is also a concept, and a fundamental on at that. It is fundamental because, terms to identify visible or perceptible objects do not require much ingenuity.

But a concept and symbol that connotes nullity represents a qualitative advancement of the human capacity of abstraction. In absence of a concept of zero there should have been only positive numerals in computation, the inclusion of zero in mathematics opened up a new dimension of negative numerals and gave a cut off point and a standard in the measurability of qualities whose extremes are as yet unkonwn to human beings, such as temperature.

In ancient Indian this numeral was used in computation, it was indicated by a dot and was termed Pujyam. Even today we use this terms for zero along with the more current term Shunyam meaning blank. But queerly the term Pujyam also means holy. Pram-Pujya is a prefix used in written communication with elders. In this case it means respected or esteemed. The reason why the term Pujya-meaning blank-came to be sanctified can only be guessed.

Indian philosophy has glorified concepts like the material world being an illusion(Maya), the act of renouncing the material world (Tyaga) and the goal of merging into the viod of eternity (Nirvana). Herein could lie the reason how the mathematical concept of zero got a philosophical connotation of reverence.

It is possible that like the technique of algebra; the concept of zero also reached the west through the Arabs. In ancient India the terms used to describe zero included Pujyam, Shunyam, Bindu the concept of a void of blank was termed as Shukla and Shubra. The Arabs refer to the zero as Siphra of Sifr from which we have the English terms Cipher or Cipher. In English the term Cipher connotes zero or any Arabic numeral. Thus, it is evident that the term Cipher is derived from the Arabic sifr which in turn is quite close the Sanskrit term Shubra.

Algebra–The Other Mathematics

In India arround the 5th century A.D. a system of mathematics than made astronomical causation easy was developed. In those times its application was limited to astronomy as its pioneers were Astronomers. As trinomial calculation are complex and involve many variables that go into the derivation of unknown quantities. Algebra is a short-hand method of calculation and by this feature, it scores over conventional arithmetic.

In Ancient India conventional mathematics termed Ganitam was known before the development of algebra. This is borne out by the name-Bijaganitam, which was given to the algebraic form of computatin. Bijaganitam means 'the other mathematics' (Bija means 'another' or 'second' and Ganitam means mathematics). The fact that this name was chosen for this system of computation implies that it was recognized as a parallel system of computation, different from the conventional one which was used since the past and was till then the only one. Some have interpreted the term Bija to mean seed, symbolizing origin of beginnings. And the inference that Bijaganitam was the original form of computation is derived. Credence is lent to this view by the existence of mathematics in the Vedic literature which was also shorthand method of computation. But whatever the origin of algebra, it is certain that this technique of computation. But whatever the origin of algebra, it is certain that this technique of computation originated in India and was current around 1500 years back, **Aryabhattta** an Indian mathematician

who lived in the 5th century A.D. has referred to Bijaganitam in his treatise on Mathematics, Aryabhattiya. An Indian mathematician-astronomer, Bhaskaracharya has also authored a treatise on this subject, the treatise which is dated around the 12th century A.D. is entitled 'Siddhanta-Shiromani' of which one section is entitled Bijaganitam.

Thus, the technique of algebraic computation was known and was developed in India in earlier times. Froom the 13 century onward, India was subject to invasions from the Arabs and other Islamised communities like the Turks and Afghans along with these invader; came chroniclers and critics like Al-beruni who studied Indian society and polity.

The Indian system of mathematics could not have escaped their attention. It was also the age of the Islamic Renaissance and the Arabs generally improved upon the arts and sciences that they imbibed from the land they overran during their great Jehad. The system of mathematics they observed in India was adapted by them and given the name 'Al-Jabr' meaning 'the reunion of broken parts.' 'Al' means 'the & Jabr' mean 'reunion.' This name given by the Arabs indicated that they took it from an external source and amalgamated it with concept about mathematics.

Between the 10th to 13th centuries, the Christian kingdoms of Europe made numerous attempts to reconquer the birthplace of Jesus Christ from its Mohammedan-Arab rules. These attempts called the Crusades failed in their military objective, but the contacts they crated between oriental and Occidental nations resulted in a massive exchange of ideas. The technique of algebra could have passed on to the west on the west at the time.

During the Renaissance in Europe, followed by the industrial revolution, the knowledge received from the east was further developed. Algebra as we know it today has lost any characteristics that betray is eastern origin save the fact that the term 'algebra' is a corruption of the term 'Al jabr' which the Arabs gave to Bijaganitam incidentally the term Bijaganit is still in use in India to refer to this subject.

In the year 1816, an Englishman by the the name James Taylor translated Bhaskara's Leelavati into English. A second English Translation Appeared in the following year (1817) by the English astronomer Henry Thomas Colebruke. Thus, the works of this Indian mathematician astronomer were made known to the western world nearly 700 years after he had penned them, although his ideas had already reached the west through the Arabs many centuries earlier.

In the words of the Australian Indologist A.L. Basham (A.L. Basham; The Wonder That was India.) "....the world owes most to Indian in the realm of mathematics, which was developed in the Gupta period to a state more advanced than that reached by any other nation of antiquity. The success of Indian mathematics was mainly due to the fact that Indians had a clear conception of the absract number as distinct from the numerical quantity of objects or palatial extension."

Thus, Indians could take their mathematics concepts to an abstract plane and with the aid of a simple numerical notation devise a rudimentary algebra as against the Greeks or the ancient Egyptians who due to their concern with the immediate measurement of physical objects remained confined to Mensuration and Geometry.

The Chief exponent of this Indo-Arab amalgam in mathematics was Al Khwarazmi who evolved a technique of calculation from Indian sources. This technique which was named by Westerners after Al Khwarazmi as "Algorismi" gave us the modern term Algorithm, which is used in computer software.

Algorithm which is a process of calculation based on decimal notation numbers. This methods was deduced by Khwarazmi's from the Indian techniques geometric computation which he has stied. Al khwarazmi's work was translated into latin under the title "De Numero Indica" which means 'of India Numerals' thus Betraying its India its Indian origin. This translation which belong to the 12th century A.D. credited to onee Adelard who lived in a town called bath in Britain.

Thus, Al Khwarazmi and adelard clould looked upon as pioneers who transmit Indian numerals to the west. Incidents according to the oxford Dictionary, world algorithm. Which we use in the English language is a corruption of the name Khwarazmi which literally means (a person) from Khwarazmi,' which was the name of the town where Alkhwarazmi lived. To day unfortunately, the original Indian texts that Al Khwarazmi studied are lost to us, only the translation are available.

The Arabs borrowed so much from India the field of mathematics that even the subject of mathematics in Arabic came to known as Hindsa which means from Indian and a mathematics or engineer be in Arabic is called Muhandis which means 'an expert in mathematics.' The word Muhandis possibly derived from the Arabic term Mathematics *Viz* Hindsa.

Contribution of Mathematicians

(A) Arya-Bhata (476 A.D.)

The Indian mathematician and astronomer Aryabhata (476 A.D) is well known for his work. 'Arya Bhatiya' dealing will astronomy and mathematics (Quadratic equation) tables of sines and other rules of algebra and trigonometry.

He was born in 476 A.D. at **Pataliputra near Patna in Bihar**. He gave the idea of represented by the first 25 (वर्ग) letters. For example—ka (क) mean 1 kha (ख) means 2 and so on. Similarly letter य, र, ल, व, श, ब, स, ह, represented 30, 40, 50, 60, 70, 80, etc. Where as अ, आ, इ, ई, उ, etc. represents 10^0, 10^1,10^2 and so on.

His most famous book is known as 'Aryabhartia'. One chapter of the book refers to mathematics while the remaining four chapter deal with Astronomy. In arithmetic, Algebra and place Geometry Aryabhata suggested humerons rules. A few important rules are enlisted Below :

1. Area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$
2. The value of Pi (π) $= 3.1416$
3. Area of circle πr^2
4. Sum (sn) of AP $= \frac{n}{2} [2a + (n-1)\, d]$

Besides this, he gave the idea of decimal system, properties of (corresponding sides are proportional). Similar triangles method of finding sum of squares and cubes of natural numbers are pythagoras theoren.

To sum up Aryabhata was really one of the greatest genius of his time in the field of mathematics and Astronomy.

(B) Brahmagupta (598-665 A.D.)

The ancient Indian astronomer brahmagupta is credited with having put forth the concept or zero for the fist time : Brahmagupta is said to have been born in the year 598 A.D. st Bhillamala (today's Bhinmal) in Gujarat, Western India. Much is known about Brahmagupta's early life, we are told that his name as a mathematician was well established when K. Vyaghramukha of the Chapa dynasty made him the court astronomer. Of his two treatises, Brahma-sputa sidhanta and Karanakhandakhadyaka, first is more famous. It was a corrected Version of the old Astronomical text, Brahma siddhanta, it was in his Brahma-sputa siddhanta, for the first time ever he had formulated the rules of the operation zero, foreshadowing the decimal system numeration. With the itegration of zero into the numerals it bacame possible to note higher numerals with limited characters.

In the earliear Roman and Babylonian systems of numeration, a large number of chart acters were required to denote higher numerals. Thus enumeration and computation became unwieldy. For instance, as E the roman system of numeration, the number thirty would have to be written as X: while as per the decimal system it would 30, further the number thirty three would be XXXIII as per Roman system, would be 33 as per the decimal system. Thus, it is clear how the introduction of the decimal system made possible the writting of numerals having a high value with limited characters. This also made computation easier.

Apart from developing the decimal system based on the incorporation of zero in enumeration, Brahmagupta also arrived at solution for indeterminate equations of 1 type ax $2+1 = y^2$ and thus can be called the founder of higher branch of mathematics called numerical analysis. Brahmagupta's treatise Brahma sputa-siddhanta was translated into Arabic under the tile Sind Hind.

For several centuries this translation gained a standard text of reference in Arab world. It was from this translation of an Indian text of mathematics that the Arab mathematics perfected the decimal system and gave the world its current system of enumeration which we call the Arab numerals, which are originally Indieal numerals.

(C) Bhaskara (1114 A.D.-1185 A.D.)

Bhaskara or Bhaskaracharya is the most well known ancient Indian mathematician. He was born on 1114 A.D. At Bijjada Bida (Bijapur,Karnataka) in the Sahyadari hill. He was the first to declare that any number divided by zero is infinity and that the sum of any number and infinity is infinity. He is famous for his book **Siddhanta Siromani (1150 A.D.).** It is divided into four sections—**Leelavati** (a book arithmetic), **Bijaganita** (algebra), **Goladnayaya** (chapter on sphere—celestial globe), and **Granoganita** (mathematics of a planets), **Leelavati** contain many interesting

problems and was a very popular text book. Bhaskara introduce **Chakrawal,** or the cyclic method, to solve algebraic expressions. Six centuries later, European mathematician like Galois, Euler and lagrange rediscovered its method and called it **"Inverse cyclic"**. Bhaskara can also be called the founder of differential calculus. He gave an example of what is now called **"differential coefficient"** and the basic idea of what is now called **"Rolle's theorem."** Unfortunately, later Indian mathematician did not take any notice of this. Five Centuries, later, Newton and Leibnitz developed this subject. As an astronomer Bhaskara is renowned for his concept of **Tatakalikagati (instantaneous motion).**

(D) Shrinivasa Ramanujan Aiyanger

Ramanujan one of the Indian mathematics (1887-1920) is best known for his work on hypergeometric series and continued fraction.

Ramanujan was born in Brahmin family on December 22, 1887 at Erode (Tanjore district) Madras. He got his school education at Kumba-koram. He won a scholarship in matriculation examination. His teachers were very much impressed by his injected and special gifted abilities in mathematics.

At the age of thirteen, he could solve all problem of Lone's trigonometry without any external help any by the time he was fourteen, he got the theorems for sine and the cosine which were given by L. Euler, Besides this in 1903 he studied the synopsis of George Schoo Bridge, related to elementary result in pure and applied mathematics.

This book opened a whole for new world him. He got the soloution with his own methods.

Later, on the he joined the government college, Kumba-konam. For the sake of mathematics, he took least interest in English. Due to this he was plucked in the examination and was debarred from scholoarship. After this he went to Vishakhapatnam (A.P) and then to Madras. Once again he tried to pass his university examination but due to ill health at the time of examination in (1907), he could not do so. He did not loose heart and worked independently for years together.

In 1909, he was married. For livelihood a means of earning was must. He got a job of clerk in the office of Madras Port Trust. Even in service here kept his interest alive in the field of mathematics.

In 1991, he started to publish some of his finding, Prof. G.H. Hardy of the university of Cambridge was greatly impressed by his talented abilities in the field of mathematics. After going through 120 theorems of Ramanujan. Professor Hardy remarked :

"I had never seen anything the least like them before. A single look at them is enough to show that they could only be written down by a mathematician of the highest class."

His work thrown light on Divergent series; Hypergeometric series and continued fraction, Definite integrals. Partition-functions, Ecliptic function, the theory of numbers, fractional differentiation and highly composite numbers.

In 1914 Ramanujan went to Cambridge. His patience, memory, power of calculation and intution made him the greatest formalist of his time. Later on, in 1918 he was elected a fellow of trinity college Cambridge. He did a lot of work in very short period of his life. Due to Tuberculosis, he died on April 26, 1920. He was simply thirty three old at that time.

"He was unquestionably one of the great masters."

(E) Blaise Pascal

He was a French mathematician, physicist, religious philosopher, and a master of prose. He laid the foundation for the modern theory of probablities. He formulated Pascal's law of pressure and propagated a religious doctrine that taught the experience of God through the heart rather than through reason. Pascal's theory of probability was based upon gambling for as he once said him self, "we are compelled to gamble." He said that if two players of equal skill were to leave the table before the game, their scores, the number of points which constitute the game being given, it is desired to find in what proportion they should divide the stakes. Through his studies he discovered that whenever a game is won by whoever obtained m + n points (one) player has m while other has n points the answer is simply derived from the arithmetical triangle.

P.C. Mahalanobis—He founded the Indian Statistical Research Institute (ISRI) in Calcutta. In 1958, he started the National Sample Surveys which gained international fame. He died in 1972 at the age of 79.

C.R. Rao—A well know statistician, famous for his "theory of estimation" (1945). His formulae and theory include "Cramer-Rao inequality", "Fischer-Rao theorem" and "Rao-Blackwellisation."

C.R. Kaprekar (1905-1988)—Fond of numbers. Well known for "Kaprekar constant" 6174. Take any four digit number in which all digits are not alike. Arrange its digits in descending order and substract from it the number by arranging the digits in ascendin order. If this process is repeated with remainders ultimately number 6174 is obtained, which then generates itself.

Harish Chandra (1923-1983)—Greatly developed the branch of higher mathematics known as the infinite dimensional group representation theory.

Narendra Karmarkar—Indian born Narendra Karmakar, working at Bell labs USA, stunned the world in 1984 with his new algorithm to solve linear programming problems. This made the complex calculations much faster, and had immediate application in airport, warehouses, communication networks, etc.

(F) Euclid (300 B.C. - 295 B.C.)

He was born in Alexadria (Sikendria), He worked as a teacher in this university for about 30 years. The ancient mathmematics was perfected and treasured in the library of this university and other libraries of the city, Euclid wrote several books. His most important works is **'Element'** a collection of thirteen books. The extend of the subject matter of the thirteen books of the **'Elements'** is :

Book I Triangles, perpendiculars, parallels, areas of rectilinear figure, Pythagoras theorems.

Book II	Transformation of areas, geometric algebra.
Book III	Circles, chords and tangents.
Book IV	Polygons and circles, construction of regular polygons including the regular pentagon.
Book V	Treatment of Proportion
Book VI	Idea of proportion applied to similar figures.
Book VII	Theory of numbers, classification of numbers as even, odd, odd times, odd plane, solid perfect etc., numerical theory of proportion.
Book VIII	Study of continued proportion.
Book IX	Number theory including a proof that the number of primes is infinite.
Book X	Study of irrationals
Book XI	Solid geometry
Book XII	The method of exaustion (used to show that circles are porportional to their diameters etc.)
Book XIII	Regular solids.

The popularity of Euclid's **'Elements'** evidenced by the fact that it was one of the first works to be translated from the Greek to various languages. It was translated into the Arabic under the patronage of Harunal-Rashid. Its translation to Latin was among the first of the translations of the twelfth century since its first version in English. Many other writings of Euclid included his dealing with astronomy, music and optics.

G. Pythagoras

The exact date and the place of his birth are unknown. Some evidences indicate that he was born between 580 and 568 B.C. It is generally believed that Pythagoras came from the island of Samos belonging to Jonian colony of Greeks planted on western shore and Island of Asia Minor. At any rate he lived in Greece at the drawn of the golden age.

Pythagoras was pupil of Thales (640 B.C. — 550 B.C) After studying under his great teacher, he travelled to Egypt, and possibly he may even have come to India. He settled in Crotona, a Greek town in south Italy, where he gathered a group of wealthy persons and founded the brotherhood of the Pythagoreans. His brotherhood has ever since served as model for all the sectret societies in Europe and America. Their badge was a five—pointed star and their studies were arithmetic, music, geometry and astronomy. Their astronomy had advanced to the point of considering that the earth was a sphere. It also led him to dwell upon the mystic properties of numbers and to consider arithmetic as one of the four degree of wisdom—arithmetic, music, geometry, and astronomy.

He is creditied to have discovered the proof of the theorem of right angled triangle. He attributed various numbers and forms to all objects for example—*'one'* with the

point, '*two*' with the line, 'three' with the surface and '*four*' with the solid. He attributed various numbers and forms of physical elements, for examples, as—'*five*' is the cause of colour, '*six*' of cold, '*seven*' of health and '*eight*' of love.

He is also credited inventing the mustical science or the harmonic canon. He is said to have discovered the harmonical progression in the notes of a musical scale, by finding the relation between the length of a string and the pitch of vibrating notes. His greatest contribution to mathematics was his inclusion of geometry as one of the liberal sciences essential to a man's education. He devoted considerable attention to the study of areas, volumes, proportions and solids. Many great scholars of the times like Plato and Zeno studied under the pythogorens.

Pythagoras studied perfect number with veneration and interest. He teavelled to Egypt with an urge to study under the Egyptian priests.

Pythagoras died in 500 B.C away from his country in exile for political reasons but very soon world realised his greatness; Later on government of Italy erected his statue in Rome to pay honour to '*The wisest and bravest of the greeks*'.

Gauss (1777-A.D - 1855 A.D.)

Karl Fraderick Gauss was born in 1777-1855 at Brunswick. His father was a mason-labourer and wanted to make his son like him. But, his mother was very aspirant from her son. Once she asked from a friend at Gauss. "*What is your view about the future of gauss*"? He replied at once, "*greatest mathematicians of Europe*".

He contributed substantially to every leading field of mathematics. His diary reveals pioneer facts, higher trigonometry and elliptic functions. He also developed the theory of surfaces with special attention to their curvature and conditions for one surface to another. The method of '*least squares*' which is indispensable at present was invented by Gauss. In 1809, he published his famous book '*Theory of Motion of Heavenly Bodies Revoluing Round the Sun in Conic Section.*' He also made many significant advances in Geometry.

Napier (1550 A.D. - 1617 A.D.)

John Napier belonged to noble Scottish family. He made a sort of chart-schemetic to which digit moved rookes and bishops on board. He propounded the binary system of writing the numbers as against decimal system. In 1950 A. D. Napier discovered Logarithm—a device which replaces multiplication by addition. He perpared the tables of logarithm which proved very practical in other branches of mathematics and sciences. The preparation of tables of logarithm was a life-long task.

Napier's treatment of the subject shows intimate knowledge of correspondence between arithmetical and geometrical progression. He was also a geometer and devised new methods in spherical astronomy.

Plato (427-347 B.C)

The great Philosopher, political thinker, mathematician, social reformer and educationist Plato was born in 427 B.C. in Athens. At the age of 20, he came in contact of another great philosopher of Athens Socrates and studied for eight year under his guidance. He contributed to mathematics in general and geometry in particular. It is

evident from many mathematical illustrations in his books. He was so enthusiastic and influenced by mathematics that he has put an inscription on his lecture room of academy which read as "*Let no one destitute of Geometry enter my doors.*"

Einstein

The great scientist of nineteenth century was born in a Jewish family in 1879 in Germany. In 1905 he mentioned the principle of relativity in his revolutionary views. In 1913, he was appointed as professor at Berlin University. In 1916, he published his **'General Principle of Relativity'**. Between 1921 and 1933 A.D., he toured and lectured all over the world and was awarded nobel prize. He died on April 11, 1955 at the age of 76. He is known as the great scientist of modern age.

17th Century Mathematics : Descartes

Rene Descartes has been dubbed the "Father of Modern Philosophy". but he was also one of the key figures in the Scientific Revolution of the 17th Century, and is sometimes considered the first of the modem school of mathematics.

As a young man, he found employment for a time as a solider (essentially as a mercenary in the pay of various forces, both Catholic and Protestant). But, after a series of dreams or visions, and after meeting in the Dutch philosopher and scientist Isaac Beeckman, who sparked his interest in mathematics and the New Physics, he concluded that his real path in life was the pursuit of true wisdom and science.

Back in France, the young Descartes soon came to the conclusion that the key to philosophy, with all its uncertainties and ambiguity, was to build it on the indisputable facts of mathematics. To pursue his rather heretical ideas further, though, he moved from he restrictions of Catholic France to the more liberal environment of the Netherlands. Where he spent most of his adult life, and where he worked on his dream of merging algebra and geometry.

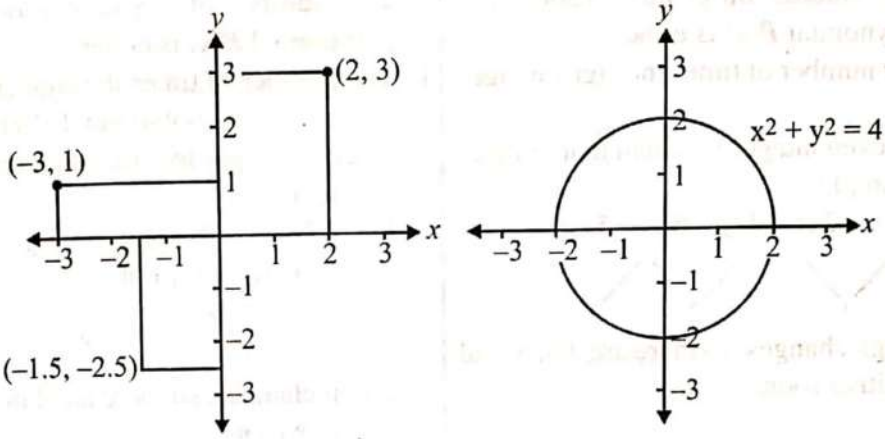
In 1637, he published his ground-breaking philosophical and mathematical treatise "Discours de la method" (the "Discourse on Method"), and one of its appendices in particular, "La Geometrie", is now considered a landmark in the history of mathematics. Following on from early movements towards the use of symbolic expressions in mathematics by Diophantus. Ai-Khwarizmi and Francois Viete, "La Geometrie" introduced what has become known as the standard algebraic notation, using lowercase a, h and c for known quantities and x, y and z for unknown quantities. It was perhaps the first book to look like a modem mathematics textbook, full of a's and b's, x^2s, etc.

It was in "La Geometrie" that Descartes first proposed that each point in two dimensions can be described by two numbers on a plane, one giving the point's horizontal location and the other the vertical location, which have come to be known as Cartesian coordinates. He used perpendicular lines (or axes), crossing at a point called the orgin, to measure the horizontal (x) and vertical (y) locations, both positive and negative, thus effectively dividing the plane up into four quadrants.

Any equation can be represented on the, plane by plotting on it the solution set of the equation. For example, the simple equation $y = x$ yields a straight line linking

together the points (0, 0) (1, 1) (2, 2) (3, 3) etc. The equation $y = 2x$ yields a straight line linking together the points (0, 0), (1, 2), (2, 4), (3, 6), etc. More complex equations involving x^2, x^3, etc. plot various types of curves on the plane.

Cartesian coordinates describe the position of a point in two dimensions by giving its horizontal and vertical locations. Thus, allowing a series of points generated by an algebraic equation to be plotted visually as a line or curves on a graph.



Cartesian Co-ordinates

As a point moves along a curve, then, its coordinates change, but an equation can be written to describe the change in the value of the coordinates at any point in the figure. Using this novel approach, it soon became clear that an equation like $x^2 + y^2 = 4$, for example, describes a circle; $y^2 - 16x$ a curve called a parabola; $x^2/a^2 + y^2/h^2 = 1$ an ellipse; $x^2/a^2 - y^2/h^2 = 1$ a hyperhola; etc.

Descartes' ground-breaking work, usually referred to as analytic geometry or Cartesian geometry, had the effect of allowing the conversion of geometry into algebra (and vice versa). Thus, a pair of simultaneous equations could now be solved either algebraically or graphically (at the intersection of two lines). It allowed the development of **Newton's** and **Leibniz's** subsequent discoveries of calculus. It also unlocked the possibility of navigating geometries of higher dimensions, impossible to physically visualize—a concept which was to become central to modem technology and physics—thus transformating mathematics forever.

Although analytic geometry was far and away Descartes' most important contribution to mathematics, he also : developed a "rule of signs" technique for determining the number of positive or negative real roots of a polynomial; "invented" (or at least popularized) the superscript notation for showing powers or exponents (*e.g.* 2^4 to show $2 \times 2 \times 2 \times 20$; and re-discovered Thabit ibn Qurra's general formula for amicable numbers, as well as the amicable pair 9,363,584 and 9,437,056 (which had also been discovered by another Islamic mathematician, Yazdi, almost a century earlier).

For all his importance in the development of modem mathematics, though, Descartes is perhaps best known today as a philosopher who espoused rationalism and

dualism. His philosophy consisted of a method of doubting everything, then rebuilding knowledge from the ground, and he is particularly known for the often-quoted statement "Cogito ergo sum" ("I think, therefore I am").

Descartes' "rule of signs" does not give the solution of a polynomial equation, but is does give information on the number of positive and negative roots of the polynominal. It says that :

The number of positive roots of a polynomial $P(x)$ *is* either :	The number of negative roots of a polynomial $P(x)$ is either :
1. The number of times the sign changes *or* 2. An even integer less than that number Example : $P(x) = 2xs - 4x_4 - 6x_3 + 5x_2 - 2$ 1 2 3 3 sign changes, so there are 3 or 1 real positive roots	1. The number of times the sign changes in th negatives polynomial $P(x)$ or 2. an even integer less than that number Example : $P(x)=2(-x)_5-4(-x)4-6(-x)_3+5(-x)_2$ $-2=-2(x)_5-4(x)_4+6(x)_3 5(x)_2-2$ 1 2 2 sign changes, so there are 2 or 0 real negative roots

Descartes notes that this rule only applies where the polynomial has a non-zero constant term, and where the polynomal is written in descending power of x.

Descartes' Rule of Signs

He also had an influential role in teh development of modem physics, a role which has been, until quite recently, generally under-appreciated and under-investigated. He provided the first distinctly modern formulation of laws of nature and a conservation principle of motion, made numerous advances in optics and the study of the reflection and refraction of light, and constructed What would become the most popular theory of planetary motion of the late 17th century. His commitment to the scientific method was met with strident opposition by the church officials of the day.

Exercise

(A) Essay Type Questions

1. Shri Ramanujan is considered as the greatest Indian mathematician of this age. Outline his contribution to the mathematics world.
2. Discuss the contribution of Arya-Bhata and Brahamagupta in the field of mathematics.
3. Write an essay on contribution of "ancient India to mathematics."
4. Discuss the historical background of mathematics. Enlist the great Indian mathematician.
5. Explain the contribution of Arya-Bhata and Srinivasa Ramanujan as a great mathematician.
6. Discuss the contribution of following Indian mathematics in the development of mathematics :

Leonhard Euler (1707 - 1783)

Leonhard Euler (1707-1783) is another one of the greatest mathematicians.

His name was found on every single person's list of top mathematicians. His father wanted Euler to follow him into the church and sent him to the University of Basel to prepare for the ministry. He entered the University in 1720, at the age of 14, first to obtain a general education before going on to more advanced studies. Euler's great potential for mathematics was soon discovered by Johann Bernoulli in private school. Leonhard Euler made decisive contributions in all areas of mathematics. He gave the world modern trigonometry. Just as Archimedes extended Euclid's geometry to marvelous heights, so Euler took marvelous advantage of the analysis of Newton and Leibniz. It is believed that he discovered the calculus of variations first, but modestly let Lagrange take the credit. He is universally regarded as the most prolific mathematician in history and the best algorist. His colleagues called him "Analysis Incarnate". Euler combined his brilliance with phenomenal concentration. After he became totally blind, he developed the first method to estimate the Moon's orbit, and he settled an arithmetic dispute involving 50 decimal places of a long convergent series.

Unit 3: Aims, Goals and Objectives of Learning Mathematics

Aims: Practical, Disciplinary, Cultural, Vocational, Social and Aesthetic - Taxonomy of Educational of objectives: cognitive, affective and psychomotor domains for teaching Mathematics - Revised Bloom's Taxonomy of Educational of objectives with specifications - General Instructional objectives: Knowledge, Understanding, Application, Skill, Interests, Attitude, Appreciation and Personality traits - Writing General Instructional objectives, specific learning outcomes and teaching points of various content areas in Mathematics.

Introduction

Aims are general and long term goals and may be common to more than one subject. Long-term goals refer to high-level aims and tend to be related to broad reasons, why a particular subject or activities are being organized or why a particular course is being taught. Aims of teaching Mathematics and see what are the different aspects that are intended to be developed while learning this subject.

Aims of teaching mathematics can be classified under the following heads:

a) Utilitarian or Practical Aim

The following are the practical aims of teaching mathematics.

- To enable the students to have clear ideas about the number concept.
- To give the individual an understanding of ideas and operations in number and quality needed in daily life.
- To enable the individual to have a clear comprehension of the way the number is applied to all measures but most particularly to those frequently used concepts such as length volume, area, weight, temperature, speed etc.
- To enable the individual to become proficient in the four fundamental operations of addition, subtraction, multiplication and division.
- To provide the basis of mathematical skills and processes, that are needed for vocational purposes.
- To enable the learner to acquire and develop mathematical skills and attitudes to meet the demands of (i) daily life (ii) future mathematical work and (iii) work in the related fields of knowledge.
- To enable the students to make appropriate approximations.
- To enable the learner to understand the concept of ratio and scale drawing, read and interpret graphs, diagrams and tables.
- To enable the individual to apply his mathematics to a wide range of problems that occurs in daily life.

b) Disciplinary Aim

The teaching of Mathematics intends to realize the following disciplinary aims.

- To provide opportunities that enable the learners to exercise and discipline mental faculties.

- To help the learner in the intelligent use of reasoning power.
- To develop constructive imagination and inventive faculties.
- To develop the character through systematic and orderly habits.
- To help the learner to be original and creative in thinking.
- To help the individual to become self-reliant and independent.

c) **Cultural Aim**

The cultural aim can be summarized as follows:

- To enable the learner to appreciate the part played by mathematics in the culture of the post and that it continues to play in the present world.
- To enable the student to appreciate the role played by mathematics in preserving and transmitting our cultural traditions.
- To enable him to appreciate various cultural arts like drawing, design making, painting, poetry, music, sculpture and architecture.
- To provide through mathematical ideas, aesthetic and intellectual enjoyment and satisfaction and to allow creative expression.
- To help the students explore creative fields such as art and architecture.
- To make the learner aware of the strengths and virtues of the culture he has inherited.
- To develop in the individual an aesthetic awareness of mathematical shapes and patterns in nature as well as the products of our civilization.

d) **Social Aims**

The important social aims of teaching mathematics are as under

- To develop in the individual and awareness of the mathematical principles and operations which will enable the individual to understand and participate in the general social and economic life of his community.
- To enable the student to understand how the methods of mathematics such as scientific, intuitive, deductive and inventive are used to investigate, interpret and to make the decision in human affairs.
- To help the pupil acquire social and moral values to lead a fruitful life in society.
- To help the pupil in the formation of social laws and social order needed for social harmony.
- To provide the pupils scientific and technological knowledge necessary for adjusting to the rapidly changing society and social life.
- To help the learner appreciate how mathematics contributes to his understanding of the natural phenomenon.
- To help the pupil interpret social and economic phenomenon.

e) **Aesthetic aim**: it is to develop their aesthetic sensibilities, meet their varying interest and helpthem in the proper utilization of their leisure time.

f) **Vocational aim**: it is to prepare them for technical and other vocations where mathematics is applied.

Meaning of Aims

Aims are general and long term goals and may be common to more than one subject. Long-term goals refer to high-level aims and tend to be related to broad reasons, why a particular subject or activities are being organized or why a particular course is being taught. Thus aims or long term goals can be regarded as expressions of strategy. Objectives are specific, immediate and expressions of strategy. While objectives are specific, immediate and attainable goals, specific to one subject, precise and clearly defined, objectives are more directly concerned with what specifically is being attempted over a relatively short period.

Meaning of Objectives

An instructional objective is a statement of the expected result. It is a description of the learning outcome that the teacher hopes will result from the instruction, whether in a lesson unit or course. It is a statement of what students should be able to do at the end of the learning period that they could not do beforehand. Thus the term „Objective" may be defined as

> ***"An objective is a point or end view of the possible achievement in terms of what a student can do when the whole educational system is directed towards educational aims."***

Thus, an objective is a part of the aim which a school can hope to achieve. Hence an objective is a narrower term when compared to an aim. It is a statement that describes what the pupil will do or be able to do towards the realization of an educational aim. When a pupil attains an objective he realizes a part of the broad aim. In other words, an objective is a statement of the terminal behavior expected of the pupils at the conclusion of a period of learning.

In other words, we can say that the objective is a statement or a form of category which suggests any kind of change. It indicates the direction of the pupil"s growth and provides the basis for the selection of evaluation procedures. Objectives provide a link between teachers, pupils, testers and parents by focusing their attention on intended outcomes of learning. Thus objectives validate the process of education. Hence objectives have the following characteristics.

- They provide direction to the activities.
- They help for the planned change.
- They provide a basis for organizing teaching-learning activities.

The objectives are classified into two categories

i. Educational Objectives: Educational Objectives are broad and philosophical in nature. They are related to the schools and educational systems. E.J Frust has well defined "Educational objective as a desired change in the behaviour of a person that we try to bring about through education". According to B.S. Bloom " Educational objectives are not only the goals towards which the curriculum is shaped and towards which the instruction is guided, but they are also the goals which provide the detailed specification of the curriculum and use

of evaluation techniques" The educational objectives are achieved with the help of teaching or instructional objectives.

ii. Teaching Objectives: Teaching objectives are narrow and psychological in nature. Teaching objectives may be achieved during a certain period in the classroom. These are related to the expected change in behavior of the child. So they are also called behavioral objectives. Teaching objectives are directly related to the learning process and they are well defined, definite, clear, specific and measurable. These give direction to the learning processes, learning-experiences and teaching. They provide the foundation of the entire educational structure. Therefore, teaching objectives are also called instructional objectives. The teaching strategies methods and techniques are selected based on teaching or instructional objectives.

Difference between Aims and Objectives:

Aims and the objectives may be compared based on the following points

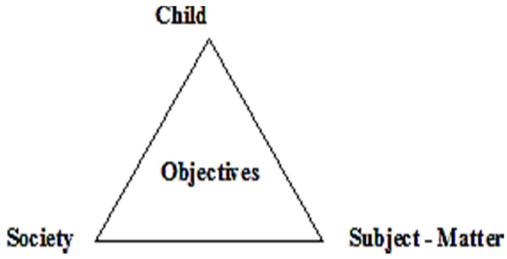
Aims	Objectives
1. Aims are very broad and comprehensive	1. Objectives are narrower and specific
2. Philosophy and sociology is the main source of aim	2. Psychology is the main source of objectives.
3. They are not definite and clear	3. They are definite and clear
4. They are difficult to achieve	4. They can be achieved conveniently
5. Long duration is required for the aims to be achieved	5. Achieved within a short duration i.e. within the classroom period
6. They are subjective	6. They are objective
7. These cannot be evaluated	7. These can be evaluated
8. These include objectives	8. Objectives are a part of the aims
9. They are related with the whole education system and whole curriculum	9. These are related with the teaching of any specific topic
10. It is the responsibility of the school, society and nation to achieve them	10. Generally teacher is responsible
11. These are theoretical and indirect	11. Objectives are direct and concerned with the teaching learning process.
12. Aims are formal	12. These are functional and informative.

TAXONOMY OF EDUCATIONAL OBJECTIVES

Based on the above clarification, objectives related to education as a whole, are known as 'educational objectives'. In the words of B.S.Bloom "By educational objectives, we mean explicit formulations of the ways in which students are expected to be changed by the educative process, that is , the ways in which they will change in their actions". In order to save time and effort, it is very important that the objectives of a subject be clearly identified and defined.

The formulation of educational objectives is a matter of careful choice on the part of the teachers and administrators. The following factors are involved:

1. The needs and capabilities of the pupils.
2. The specific demands of his social environment.
3. The nature of the subject matter.



The criteria of a good educational objective is that it is,

1. In accordance with general aims of education
2. Unambiguous
3. Useful
4. Specific and
5. Feasible

CLASSIFICATION OF EDUCATIONAL OBJECTIVES

Educational objectives indicate the nature of the education system and show the direction in which education will act. Educational objectives serve as guides for teaching and learning. These also develop awareness among the teachers about the importance of their work and provide guidelines in selecting teaching-learning activities. For teaching which aims at worthwhile behaviour changes, a clear understanding of educational objectives is essential.

Some educators have attempted classification of educational objectives. Classification is a valuable system to group similar things under one heading based on common characteristics or common relationship that exists between groups and individuals. **The main functions of classifying educational objectives are as follows.**

1. It is helpful in planning curriculum.
2. It is helpful in planning, teaching and learning activities.
3. It is helpful in identifying desired behavioural outcomes among the learners.
4. It is helpful in preparing evaluation or testing materials.
5. It is helpful in comparing curricular goals with wider educational objectives.
6. It is helpful in the search for the relationships that exist among groups and individuals.
7. It is helpful in defining, translating and exchanging educational thoughts in a uniform way.

BLOOM'S TAXONOMY

A number of attempts have been made by experts in the field of classification of educational objectives. Perhaps the most widely used system of classification of objectives is the one prepared by a group of college and university teachers and later reported in "Taxonomy of Educational Objectives" (1956) edited by Benjamin S. Bloom. 'Taxonomy' is a term derived from two Greek words 'taxis' meaning 'arrangement' and 'nomos' meaning 'law'. In this derivative sense, taxonomy means 'orderly arrangement'. They classified the educational objectives into three broad categories or domains. The Domains are:

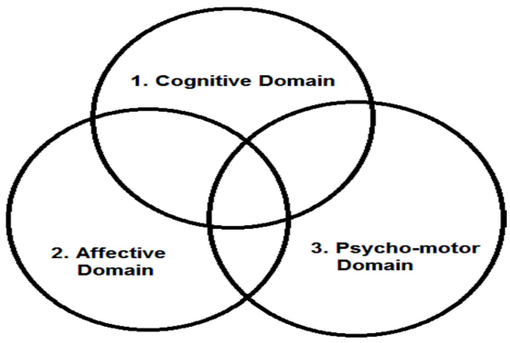
a. Cognitive Domain (knowing)
b. Affective Domain (feeling)
c. Psycho-Motor Domain (doing)

These domains are hierarchical because when learning takes place, the first activity is a mental process of trying to understand, analyze, synthesize and associate the information with

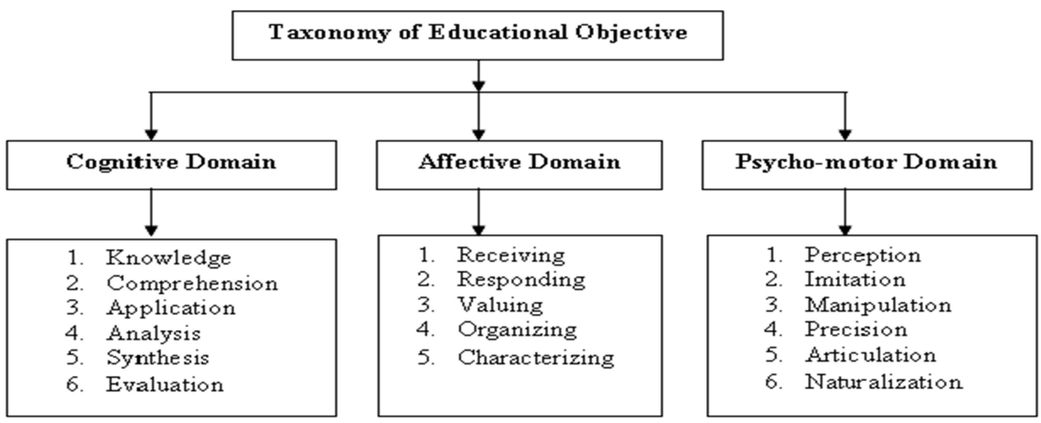
something already known. This thought process comes under cognitive domain.

In the second stage, the knowledge acquired begins to produce changes in the interest, attitude and feelings of the individual. These types of behavioural changes are grouped under affective domain. In the third stage, the feeling acquired gives way to exhibit muscular skills and manipulative skills. These changes are grouped under psycho-motor domain.

Besides, the above classification, each domain can be further split up into major categories which are also hierarchical. The major categories are as follows.



Bloom's Taxonomy



CATEGORIES IN THE COGNITIVE DOMAIN

Cognitive Objectives: Cognitive Objectives stress that the pupils should acquire more and more knowledge. It was defined to include all those activities which deal with the recall or recognition of knowledge and the development of intellectual abilities and skills. The arrangement in this category can be observed as follows.

Knowledge

This is the first and the lowest level of cognitive domain. It includes recall of information such as specifications, facts, methods, processes, generalizations, patterns etc., and thus the knowledge objective emphasizes what can be described as memory.

Comprehension
This second category includes Translation, Interpretation and Extrapolation. This is also related to the use of ideas. It refers to a type of understanding of the materials or literal message contained in a communication.

Application
The third level includes the ability to apply abstract ideas to a concrete situation. The abstractions may be in the form of general ideas, rules or procedures or generalized methods.

Analysis
It means the "break down of the materials into its constituent parts and detection of the relationships of the parts and of the way they are organized. "Analysis includes analysis of elements, analysis of relationships and analysis of organizational principles.

Synthesis
This category is just the opposite of analysis. Synthesis is the "Putting together of elements and parts so as to form a whole. This involves the process of working with pieces, parts, elements and arranging and combining them in such a way as to constitute a pattern or structure not clearly there before".

Evaluation
It is the assignment of symbols to phenomenon, in order to characterize the worth or value of a phenomenon, usually with reference to some social, cultural or scientific standards. Evaluation involves judgments in terms of internal evidence as well as external criteria.

To conclude, it may be pointed out that the above six major categories in the cognitive domain do not always appear in the isolation from one another.

CATEGORIES IN THE AFFECTIVE DOMAIN

Affective Objective: Affective objective is concerned with the attitude, interest, emotions, values and mental tendencies of the pupils. This part of the taxonomy also includes appreciations and social adjustment of the child. The arrangement in this category can be observed as follows.

Receiving
This is at the lowest point on the affective domain. Receiving may be defined as "Sensitivity to the existence of certain phenomena and stimuli, that is, the willingness to receive or attend to them".

Responding
Responding refers to a behaviour which goes beyond merely attending to the phenomena; it implies active attending, doing something with or about the phenomena, and not merely perceiving them.

Valuing
Valuing implies "Perceiving them as having worth or value. The three sub- categories of this objective are acceptance of value, preference for a value commitment.

Organising
This involves building up of organized system of values. The individual organizes a set of values such as truth, goodness and helping others in determining their relationships and deciding their need and priority.

Characterising
In this category, the individual displays the integration of values and it becomes a lifestyle with him. He gets these values organized into some kind of internally consistent system, which has controlled the behaviour of the individual for a sufficient time. This category is concerned with one's view of the universe and one is philosophy of life.

CATEGORIES IN THE PSYCHO-MOTOR DOMAIN

Psychomotor Objective: This third part of taxonomy and includes the manipulative and motor-skill areas. The physical actions involved in handwriting, playing, using equipments, making an outline, drawing figures and many others are in the psycho-motor domain. The arrangement in this category can be observed as follows.

Perception
Skill of keen observation, skill of sensing a problem and skill of developing self- motivation are the specific objectives under this category.

Imitation
Skill of repeating the actions and skill of reflective thinking are the specific objectives under this category

Manipulation
Skill of operating upon the intelligence and manage cleverly by using unfair method are the specific activities that fall in the category.

Precision
Skill of experimentation, skill of précised movements and neat execution of skills are the activities that fall under this objective.
Articulation
Skill of global thinking, reflective thinking, skill of mind and body and the development of mathematical skills are specific objectives of this step.

Naturalization
As we practice a skill, in due course it becomes our natural habit. Skill of attaining success and skill of multiple actions are the specific activities under this objective.

Since science is a study usually involving direct experimentation, the psycho- motor domain has got great relevance.

Revised bloom taxonomy

The original Bloom's educational taxonomy Bloom et. al (1956) published the taxonomy of educational objectives: a cognitive domain. Bloom et. al (1956) classified forms and levels of learning based on cognitive processes that learners involved in when they learn. Bloom considered his initial effort to be a starting point, as evidenced in memorandum from 1971 in which he stated "Ideally each major field should have its own taxonomy in its own language – more detailed, closer to the special language and thinking of its experts, reflecting its own appropriate sub-division and levels of education, with possible new categories, combinations of categories and omitting categories as appropriate"

Anderson's Revised taxonomy as a match to Bloom's taxonomy Anderson (1990), a former student of Bloom, updated and revised the taxonomy reflecting relevance to 21st century work for both students and teachers as she said (Anderson& Krathwohl, 2001). Anderson changed the taxonomy in three broad categories: terminology, structure and emphasis (Forehands, 2005). Anderson modified the original terminology by changing Bloom's categories from nouns to verbs. Anderson renamed the knowledge category into remember, comprehension into understanding and synthesis into create categories. Anderson also changed the order of synthesis and placed it at the top of the triangle under the name of Create (Taylor & Francis, 2002). Thus, Anderson and Krathwohl's (2001) revised Bloom's taxonomy became: Remember, Understand, Apply, Analyze, Evaluate and Create (Figure 1).

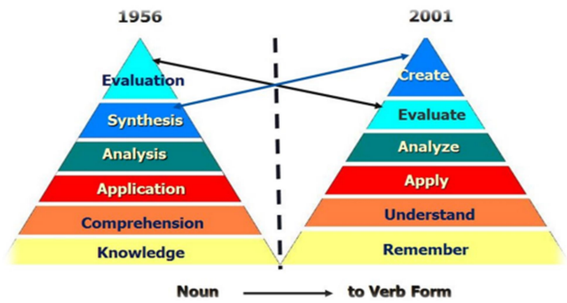


Figure 1. The Revised Bloom's Taxonomy by Anderson and Krathwohl (2001).

Anderson and Krathwohl (2001) also made structural changes to the original Bloom's taxonomy. Anderson considered two dimensions in the revised taxonomy instead of one, a products dimension. The two dimensions are: 1) knowledge (or the kind of knowledge to be learned) and 2) cognitive process (or the cognitive processes to be used in acquiring knowledge). The intersection of the knowledge and cognitive categories form 24 separate cells as represented in Figure 2. Based on Anderson's perspective, the Knowledge Dimension on the left side is composed of four kinds: Factual, Conceptual, Procedural, and Meta-Cognitive knowledge. The Cognitive Process Dimension across the top of the grid consists of six levels: Remember, Understand, Apply, Analyze, Evaluate, and Create

	The Cognitive Process Dimension					
	Remember	Understand	Apply	Analyze	Evaluate	Create
Factual Knowledge	List	Summarize	Classify	Order	Rank	Compile
Conceptual Knowledge	Describe	Interpret	Experiment	Explain	Assess	Plan
Procedural Knowledge	Tabulate	Predict	Calculate	Differentiate	Conclude	Compose
Meta-Cognitive Knowledge	Appropriate use	Execute	Construct	Achieve	Action	Actualize

Figure 2. Knowledge and Cognitive Dimensions of Bloom's Taxonomy as revised by Anderson.

Objectives of Teaching Mathematics

The objectives of teaching mathematics at the secondary stage may be classified as under:

- Knowledge and Understanding objectives
- Skill objectives
- Application objectives
- Attitude objectives
- Appreciation and
- Interest objectives and
- Personality traits

A. Knowledge and Understanding Objectives

The student acquires knowledge and understanding of:

- Language of mathematics i.e., the language of its technical terms, symbols, statements, formulae, definitions, logic, etc.
- Various concepts i.e., the concept of number, the concept of direction, concept measurement.
- Mathematical Ideas, like facts, principles, processes and relationships.
- The development of the subject over the centuries and contributions mathematicians.
- Inter-relationship between different branches and topics of mathematics etc.
- The nature of the subject of mathematics.

B. Skill Objectives

The subject helps the student to develop the following skills:

- He acquires and develops skills in the use and understanding of mathematical language.
- He acquires and develops speed, neatness, accuracy, brevity and precision in mathematical calculations.
- Learns and develops the technique of problem-solving.
- Develops and ability to estimate, check and verify results.
- Develops and ability to perform calculations orally and mentally.
- Develops and ability to think correctly, to draw conclusions, generalizations and inferences.
- Develops skills to use mathematical tools, and apparatus.
- Develops essential skills in drawing geometrical figures.
- Develops skills in drawing, reading, interpreting graphs and statistical tables.
- Develops skills in measuring, weighing and surveying.
- Develops skill in the use of mathematical tables and ready references.

C. Application Objectives The subject helps the student to apply the above-mentioned knowledge and skills in the following way:

- Able to solve mathematical problems independently.
- Makes use of mathematical concepts and processes in everyday life.
- Develops ability to analyze, to draw inferences, and to generalize from the collected data and evidence.
- Can think and express precisely, exactly, and systematically by making proper use of mathematical language.
- Develops the ability to use mathematical knowledge in the learning of other subjects especially sciences.
- Develops the students‟ ability to apply mathematical in his future vocational life.

D. Attitude Objectives

The subject helps to develop the following attitudes:

- The student learns to analyze the problems.
- Develops the habit of systematic thinking and objective reasoning.
- Develops heuristic attitude and tries to discover solutions and proofs with his independent efforts.
- Tries to collect enough evidence for drawing inferences, conclusions and generalizations.
- Recognizes the adequacy or inadequacy of given data in relation to any problem.
- Verifies his results.
- Understands and appreciates logical, critical and independent thinking in others.
- Expresses his opinions precisely, accurately, logically and objectively without any biases and prejudices.
- Develops self-confidence for solving mathematical problems.
- Develops personal qualities namely, regularity, honesty, objectivity, neatness and truthfulness.
- Develops mathematical perspective and outlook for observing the realm of nature and society.

E. Appreciation and Interest Objectives The student is helped in the acquisition of appreciation and interest in the following way:

- Appreciates the role of mathematics in everyday life.
- Appreciates the role of mathematics in understanding his environment.
- Appreciates mathematics as the science of all sciences and art of all arts.
- Appreciates the contribution made by mathematics in the development of civilization and culture.
- Appreciates the contribution of mathematics to field and other branches.
- Develops the interest in the learning of the subject.

- Feels enter by mathematical recreations.
- Develops interest in the activities of mathematics clues.
- Develops interest in the active library reading, mathematical projector.
- Appreciates the aesthetic nature of mathematics by observing symmetry, similarity, order and arrangement in mathematical facts, principles and processes.
- Appreciates the contribution of mathematics in the development of other branches of knowledge.
- Appreciates the recreational values of the subject and learn to utilize it in his leisure time.
- Appreciates the vocational value of mathematics.
- Appreciates the role of mathematical language, graphs and tables in giving precision and accuracy to his expression.
- Appreciates the power of computation developed through the subject.
- Appreciates the role of mathematics in developing his power of acquiring knowledge.
- Appreciates mathematical problems, their intricacies and difficulties.

F. Interest Objectives The student is helped in the acquisition of interest in the following way:

- He develops interest in the learning of the subject.
- He feels entertained by mathematical recreations.
- He takes an active interest in the activities of the mathematics club.
- He takes an active interest in the active library reading, mathematical projects, and doing practical work in the mathematics laboratory

G. Personality Traits

The teaching of mathematical science aims to develop personality traits also. The pupil should report his results and observations faithfully. Mathematical Science teaching should provide suitable opportunities for every individual to unfold environment, must be provided to draw out the best in the child-body, mind and soul.

Writing Learning Objectives and Teaching Points of Various Content Areas in Mathematics like Algebra, Geometry, and Trigonometry etc.

Introduction

Now that have understood what are the Aims and Objectives of teaching Mathematics it is necessary to look at it in a little more detailed manner. The details of objectives lie in how clearly it states what the outcome of learning is. As the major goal of every teaching activity is the optimum level of learning, one needs to be clear about what does that optimum level consist of. This clarity can be found only when the objectives are stated with a focus on learning. Such objectives which has its focus on learning and which states learning in its behavioral forms comes to be called learning objectives. In this unit, we shall discuss how to write learning objectives in Mathematics.

A. Writing Learning Objectives: A Learning Objective is an objective that describes what students should know or be able to do at the end of the course. In other words, the objectives

state the specific outcomes of learning that a student achieves when an Instructional Objective has been achieved.

Hence for each General Instructional Objective, it is necessary to write Specific Outcomes of Learning that will state the precise behaviour or performance that is expected of a student. Each general instructional objective can have many specific outcomes of learning under it. These are smaller units of performance and can be precisely measured by tests of various kinds. There are five elements which when used in writing a Specific Outcome of Learning give the clearest definition for student performance that can be used for both teaching and testing. The five elements are as follows

a) Performer (The student, the trainer, the learner, etc.)
b) Action Required (An action verb, Example: identifies, compares, describes, distinguishes, analyses, classifies, etc.)
c) Task (Include a task to be performed. Example: Compares the properties, explains the derivation.)
d) Conditions (Include any condition that may be required. Example: Compares the properties of the given triangles.)
e) Criteria for judgement (Any relevant criteria for clarity. Example: Explains the phenomena with at least two examples, computers with speed and accuracy.)

Three Qualities to be Maintained- In writing an SOL three qualities must be maintained, if SOL is to serve the purpose of communication between teacher, pupil and examiner.

- Use clear, precise action verbs.
- Must be feasible in terms of student's level, nature of the content and learning experiences.
- Must be observable and measurable.

B. Instructional Objective in Mathematics

I. Remembering: the pupil acquires knowledge of terms, concepts, symbols, definition, principles, process and formulae of mathematics at the secondary stage. It is cognitive level.

Specification: The pupil –

1. Recalls the mathematical laws, principle, rule formulae, etc.
2. Recognizes the mathematical laws, principle, formulae, etc.

II. Understanding: the pupil develops understandings of terms, concepts, symbols, definition, principles, process and formulae of mathematics at the secondary stage. Goes deep into the content

Specification: The pupil –

- Cites or gives examples.
- Gives reasons.
- Identifies.
- Compares.

- Finds relationship.
- Based on observation draw conclusions.
- Draws inference or the result.
- Converts verbal form to symbolic form or vice versa.
- Classifies mathematical data.

III. Applying: The pupil applies his knowledge and understanding of mathematics to unfamiliar situations (or new problems).

Specification: The pupil –

- Analyses the problem into its components.
- Judges the adequacy of the given data.
- Suggests the alternate methods.
- Suggest the most appropriate method.
- Analyses and finds out what is required.
- Finds out the adequacy, superfluity or relevancy of data.
- Establishes relationship among the data.
- Select the appropriate method for solving the problem.
- Suggests alternative methods.
- Generalizes *(i.e.,* reasons inductively)
- Infers *(i.e.* reasons deductively)

IV. Skill: To acquire skills of computation, drawing geometric figures and grapes reaching tables, charts, graphs etc.

Specification: The Pupil –

- Reads mathematical figures, statements, problems, charts, tables, etc.
- Labels the geometrical figure.
- Draws the most appropriate, neat and proportionate geometrical figures.
- Solves oral problems quickly and accurately.
- Solves written problems quickly and accurately.

a). computation

Specification: The Pupil –

- Carries out oral calculation with ease and proficiency
- Carries out written calculation with ease and speed

b). drawings of geometrical figures and graphs

Specification: The Pupil –

- Handles geometrical instruments with ease and proficiency
- Measures accurately
- Draws free hand figures with ease.
- Draws figures to specifications or to scale.

- Draws figures accurately.

c). reading tables, charts, graphs etc

Specification: The Pupil –

- Read tables with speed and accuracy.
- Interprets graphs

C. Learning Objectives in Mathematics

1. Remembering: The pupil acquires knowledge of mathematics.

Learning outcomes: The pupil -

- Recalls mathematical terms, facts, processes, principles, formulae definitions, signs and symbols, relationships, generalizations etc.,
- Recognizes terms, instruments, process, formulae, signs and symbols, relationships, generalisations etc.,

2. Understanding: The pupil develops an understanding of mathematics.

Learning Outcomes: The pupil -

- Explains mathematical terms, concepts, principles, etc., in his own words.
- Defines mathematical terms and concepts.
- State mathematical principles, relationships etc.
- Gives illustrations for mathematical concepts, principles, etc.,
- Identifies mathematical terms, concepts, relationships, figures, processes etc.
- Finds similarities between mathematical terms, concepts, relationships, figures etc.
- Differentiates between mathematical terms, concepts, relationships, figures etc.
- Classifies mathematical terms, concepts, figures etc.
- Verbalises symbolic relationships and vice versa.
- Frames mathematical formulae, generalisations based on data.
- Uses the formula to solve problems.
- Substitute relevant numbers, symbols and signs in the mathematical formulae and operations.
- Calculate the answers for given problems.
- Uses appropriate units to write answers.
- Finds solutions for given problems.

3. Applying: The pupil applies knowledge of mathematics to novel situations.

Learning outcomes: The pupil -

- Analyses a problem or data into component parts.
- Judges the adequacy, inadequacy or superfluity of data.
- Establishes relationships among data.
- Gives a number of methods of solving a problem.

- Select the most appropriate formulae or principles or methods or processes to solve problem.
- Reasons deductively.
- Reasons inductively.
- Makes a generalization.
- Draws inferences.
- Predicts results based on data.

4. Skill:

a) The pupil acquires skills in handling mathematical instruments with ease.

Learning Outcomes: The pupil -

- Draws freely satisfactory free-hand figures.
- Selects the most appropriate mathematical instruments.
- Takes necessary precautions in taking measurements while constructing geometrical figures.
- Takes measurements correctly.

b) Drawing geometrical figures and Graphs

Learning Outcomes: The pupil -

- Draws figures to given specifications.
- Draws figures quickly.
- Uses appropriate marking to denote different parts of a figure.

c) Computation:

Learning Outcomes: The pupil -

- Does oral calculation correctly.
- Does oral calculation quickly.
- Does a written calculation correctly.
- Does a written calculation quickly.
- Uses correct notations and symbols.
- Avoids unnecessary steps in the solution of a problem.
- Is systematic in working on a problem.

d) Reading of Tables, Charts, Graphics, etc.

Learning Outcomes: The pupil -

- Selects appropriate mathematical tables.
- Uses mathematical tables, charts, ready reckoners etc., correctly.
- Co-ordinates the different sections of the graphs correctly.
- Reads a graph correctly.

Teaching points of various content areas in Mathematics (Objectives of Teaching Arithmetic, Algebra, Geometry and Trigonometry)

A. Objectives of Teaching Arithmetic

a) Understanding Arithmetic for selecting Teaching Points

Arithmetic is a branch of mathematics that consists of the study of numbers especially the properties of the traditional operations on them- addition, subtraction, multiplication and division. Arithmetic is an elementary part of number theory and number theory is considered to be one of the top-level divisions of modern mathematics along with algebra, geometry and analysis. The terms arithmetic and higher arithmetic were used until the beginning of the 20th century as synonyms for number theory and are sometimes still used to refer to a wider part of number theory.

Arithmetic is the science of numbers and the art of computation. Historically arithmetic was developed out of a need for a system of counting. It is considered to be essential for efficient and successful living. That is why arithmetic is divided as the science that deals with numbers with relations between numbers, numbers in term, or abstraction arising from such concrete situations as counting measuring and ordering the various quantities and objects that we encounter in everyday life. The teaching of arithmetic has to fulfill two responsibilities.

- The inculcation of an appreciative understanding of the number system and an intelligent proficiency in its fundamental process.
- The socialization of number experiences.

b) Objectives of teaching arithmetic

- To develop fundamental arithmetic concepts like the concept of number, order, units of measurement, size, shape etc among the pupils.
- To train the pupils in mathematical thinking i.e. to understand the statement to analyze it and to arrive at the right conclusions.
- To arouse pupil's interest in the quantitative side of the world around them and its use as a simple tool in business.
- To develop accuracy and facilitate the simple computation of the fundamental process among pupils.
- To develop speed and accuracy in arithmetical calculation and computation among pupils
- To impart to pupils a working knowledge of practical arithmetical applications, which are useful in life.
- To develop a sense of appreciation among the pupils for the use of arithmetic in daily life.
- To prepare pupils to learn other branches of mathematics and also pursue higher studies in mathematics.

B. Objectives of Teaching Algebra

a) Understanding Algebra for selecting Teaching Points

Algebra in its most general form is the study of mathematical symbols and the rules for manipulating these symbols. It is a unifying thread of almost all of mathematics. It includes everything from elementary equation solving to the study of abstractions such as groups, rings and fields. Algebra is called the science of letter. It refers to the methods of reasoning about numbers by employing letters to represent their relationship. Algebra is concerned largely with the structure of the number system, operations with numbers and statements involving numbers as well as the solution of problems. Algebra is a language used to develop and express much of the scientific data. Algebra comprehended a more general treatment of numbers and number relations than this arithmetic. It is concerned with the general statement about numerical situations. Algebra refers to the operation of taking a quantity from one side of the equation to another by changing its signs. It presents a radically new and different approach to the study of quantitative relationships characterized by a new symbolism, new concepts, and a new language much higher degree of generalization and abstraction than has been encountered in arithmetic. But it is primarily taught for manipulative skills. Solutions of problems by equations give power of generalization and use of formulae and idea of functionality.

b) Objectives of teaching Algebra -

- To develop among pupils the skill of identifying patterns
- To develop the skill of representing real-world situations using expressions of equations.
- To develop the skill of simplifying expressions using order of operations.
- To develop among pupils the skill of solving multi-step equations.
- To develop the skill of converting equations to graph.
- To provide an effective way of expressing complicated relations.
- To inculcate the power of analysis.
- To verify the results more simply and satisfactorily.
- To provide a new and refined approach in the study of abstract mathematical relationships through the use of new symbolism.

C. Objectives of Teaching Geometry:

a) Understanding Geometry for selecting Teaching Points

The word geometry originally means the measurement of earth. It is the science of lines and figures it is the science of space and extent. It deals with the position, space and size of bodies but nothing to do with their material properties. Geometry has two important aspects – Demonstrative Geometry – It deals with the shape, size and position of figures by pure reasoning based on definitions, self-evident truths and assumptions. Euclid, a great Greek Mathematician was the father of demonstrative Geometry. His methods are intuitional, observational, intentional, constructive, informal, creative, and experimental and so on.

Practical Geometry – It covers the constructional work of the subject. Most of the work is directly or indirectly based on demonstrative Geometry.

b) Objective of teaching Geometry

- To enable the learner to acquire the knowledge of geometrical facts.
- To implement geometrical principles like equality, symmetry similarity in every nature of things.
- To develop the ability to draw accurate figures.
- To demonstrate the nature and the power of pure reason.
- To systematize the information received by the pupils in the pre-school stage.
- To aid the pupils in becoming familiar with the basic geometrical concepts and space perceptions and in understanding the fundamental techniques such as the use of set square, protractor, compass, etc.
- To acquaint the pupil with the geometrical notations.

D. Objectives of Teaching Trigonometry

a) Understanding Geometry for selecting Teaching Points

Trigonometry is a branch of mathematics that studies relationships between side lengths and angles of triangles. Trigonometry is found throughout geometry, as every straight-sided shape may be broken into a collection of triangles. Further still trigonometry has astoundingly intricate relationships to other branches of mathematics, in particular complex numbers, infinite series, logarithms and calculus. The word trigonometry is a 16th century Latin derivative from the Greek words for triangle (trigonon) and measure (metron).

b) Objectives of Teaching Trigonometry the students will be able,

- To understand trigonometric ratios and identities.
- Find the value of trigonometric ratios of some specific angles.
- Determine the trigonometric ratios of complementary angle.
- Apply the trigonometric identities in proving the given statement.
- To apply the knowledge of trigonometry to solve daily life problems.
- To find heights and distances.
- To appreciate the use of trigonometry to solve problems.
- To develop creative thinking and reasoning.
- To appreciate its usefulness in technology
- To understand the relationship between trigonometry and other branches of mathematics.

Use of Objectives in Teaching and Testing

Instruction involves three distinct kinds of activities: teaching, learning and evaluation. If the instructional process is to be effective, all three activities must be oriented to certain common objectives. The objectives form the pivot of the entire teaching-learning process. Objectives should be stated for each course, unit and topic. They are the mental skills that students should develop as a result of teaching. The objectives direct the pupils as to what he or she is expected to do, what should be the minimum level of acceptance for his or her performance and under what conditions it will be achieved. After objectives have been stated the teacher plans the teaching and learning experiences needed for the students to attain the objectives. Learning experiences are those activities that are planned with a specific purpose of bringing about the desired changes in the behavior of the students. Realization of the objectives, to a great extent, depends upon careful selection and planning of appropriate learning experiences. These experiences may include seminars, laboratory work, discussions, audiovisual presentations, research paper writing, projects etc. These techniques provide active student's involvement in learning and different techniques work for different learning needs. True learning is not merely the acquisition of knowledge or certain skills; it is a change in behavior brought about by training or experience. The changes the learner experiences are the direct outcome of his interaction with the learning environment. The teacher then selects an appropriate evaluation technique to assess the student‟s performances in terms of whether or not the students have attained the objectives described. This evaluation helps the teacher in ascertaining.

a. Whether or not the desired changes have taken place in the pupil's behavior.
b. The attainability and feasibility of the stated objectives.
c. The effectiveness of the learning experiences provided.

If the teacher is not satisfied with the outcome of the evaluation, the teacher has to critically appraise all the three components of the teaching namely, objectives, learning experiences and evaluation techniques and accordingly change or modify any of these three components.

General Instructional Objectives (GIO) and Specific Outcomes of Learning (SOL):

General Instructional Objectives (GIO) The General Instructional Objectives are for a course and can apply to any item of the curriculum/syllabus. They are intended to assist in defining and carrying out broad educational aims. By specifically stating the kind of outcome of student learning desired, these objectives can be used to clarify teaching methods, learning experiences and materials needed for particular content and course. Example:

- The pupil acquires knowledge of mathematical terms, facts, concepts, principles, theorems etc.
- The pupils understand the meaning of mathematical terms, facts etc.
- The pupil applies mathematical principles to new and unfamiliar situations.

These objectives are also known as non-behavioral objectives as they do not depict the overt behaviour of the student. The statement of such objectives contains verbs like "knows",

"understands" etc. The pupil understands the polynomial division or the pupil knows the place values of binary numbers are examples for non-behavioral objectives as „understanding" by the student or „acquisition of knowledge" by the student cannot be observed by the teacher and therefore they are not observable behaviors.

Specific Outcomes of Learning (SOL) Specific Outcomes of Learning consists of statements defining the specific performances, which we adopt as evidence that a student has actually reached his objective, all of which are precise and measurable. These objectives are also known as behavioral objectives as the statements of these objectives contain an action verb that displays an overt behaviour of the learner. Pupil explains the polynomial division or the pupil states the properties of binary addition are behavioral objectives, an explanation by the student or stating by the student are observable behaviors. There will be a large number of these objectives from which a sample may be taken as an indication that the student has attained the broader General Instructional Objectives. Example:

- The pupils recall definitions of mathematical terms or concepts
- The pupil recognizes mathematical symbols
- The pupil lists properties of geometrical figures
- The pupil classifies geometrical figures
- The pupil gives reason for mathematical statements
- The pupil establishes the relationship among mathematical concepts.
- The pupil formulates a hypothesis for solving a given problem
- The pupil selects principles relevant to the problem presented.

Criteria for Judging Instructional Objectives In choosing general instructional objectives, it is helpful to have criteria against which to judge whether or not the objectives are relevant and useful.

- Attainability – Within the realm (domination) of possibility.
- Validity – In line with the aims of education
- Comprehensiveness – Covering all the behaviors and content material
- Precision – Clear and unambiguous
- Feasibility – For application
- Appropriateness – For yielding specific outcomes
- Reasonable in number
- Consistent with one another.

Need for establishing General Objectives for teaching Mathematics

a) Makes clear what a student should be able to do on completing the course: General Objectives are stated for the collective outcome of a course and hence they make clear what a student is capable of doing at the end of that course. This includes the attitude toward mathematics, skills related to mathematics, operations and relations, values and on the whole the syllabus stipulated to that course.

b) Helps in identifying the learning outcomes and specific outcomes of learning: To identify the learning outcomes and express them as specific outcomes of learning one needs a framework. General Objectives provides this framework.

c) Guides the teacher to select appropriate methods and approaches of teaching: It is very essential to select the right methods and approaches to teaching mathematics if the learning has to be optimum. The general objectives guide the teacher in selecting them.

d) Provides the frame of reference for decisions about selection and organization of the subject matter, the mode of instruction and techniques of evaluation: General objectives make us know what is suitable for a particular course and what is expected of that course. Hence a teacher will have a perfect view of what and how to teach and evaluate.

e) Directs pupils as to what he or she is expected to achieve by the end of instruction: General Objectives also help the pupils to have an idea as to what is expected of them by the end of the course. This gives them a direction in which they have to proceed.

f) Helps the teacher to select the appropriate evaluation technique: Evaluation forms an important part of any course. And since the general objectives specify what is to be learnt during the course, it guides in selecting the right evaluation techniques which are necessary to evaluate the outcomes.

LESSON PLAN

"Lesson-plan is the outline of the important points of a lesson arranged in order in which they are to be presented to students by the teacher." —*International Dictionary of Education*

We need planning for doing any work properly. We also need proper planning for teaching in the class, because lesson planning is necessary for a successful teaching. As clear as arranged the planning is, the same is executed clearly. Lesson plan is also very important to strengthen the favour of arrangement of the whole teaching process. The lesson plan takes birth before the teacher enters the class. Due to which the teacher can perform the teaching work in the class without hesitation.

For example, it is already said that when, what and how? much does a sugar product seller drops in the sugar product before manufacturing it. Just after that, he becomes successful in his work. In the same way, an efficient teacher confirms before beginning the lesson, when and where has he to use chart, model, teaching method etc, then he achieves success in his teaching work. Due to this reason, it is necessary to determine its form before performing any work.

Origin of Lesson Plan

The origin of lesson plan is from Gestalt psychology. Gestalt theory of learning has a great influence of human learning. According to this theory, the learner perceives a thing or a problem or a situation in totality or as a whole. So, a unit plays an important role in the learning of student because he usually takes the help of units in understanding the whole concept. Thus, the whole is perceived by a part and part conveys the whole. The meaningful activities are related to one another within a unit. These activities provide the purposeful learning experiences and the learner understands the whole concept. This theory originates the concepts of unit plan.

There are two approaches of unit plan. The first approach is propounded by **Herbert.** He stresses on the content and informations in a unit plan. The second approach is given by **John Dewy and Kilpatrick.** They have emphasized on the experiences of learners in unit plan rather than information.

B.F. Skinner has provided a recent approach of unit plan. The focus of his unit plan is the modification learn better if the content is presented in small unit. The unit-plan is the crucial aspect of a lesson-plan.

Meaning of Lesson-Plan

The means of achieving a definite objective or aim of teaching work systematically and successfully is called **lesson-planning**. When any work is performed under a plan, then it is successful surely. It can be said in this way that the lesson-plan is that peredetermining plan according to which a teacher presents before the students through new knowledge methods and teaching aids in a definite time. In this way, lesson-planning is the designing of behavioural shape of all sides of teaching arrangement.

The lesson-planning takes birth during pre-stage (first stage of teaching). In this period, the teacher prepares a sketch of his all teaching activities and the teacher gives behavioural shape to planned teaching activities in the lesson planning, at the time of last activity period of teaching. The teacher arranges his teaching work in the lesson-planning what he has to teach?

Therefore, lesson-planning is an essential pre-requisite for good teaching, the structure of the plans should vary with different teaching learning situations and with the needs of different groups of pupils. More over, whatever way you plan a lesson, lesson planning is an essential part of the teacher's work. If the teacher has planned his lesson both wisely and too well he wili enter his class with confidence and with an easy conscience. However, the lesson plan is a good servant but a bad master. It is a means to an end not an end by itself. The teacher should be able to discard his plan if a sudden situation demands it.

Definitions of Lesson Plan

Main definitions of lesson planning given by different mathematicians are as follows :

According to **Bossing,** "Lesson plan is actually a plan of action."

According to **Bining and Bining,** "Daily lesson planning involves defining the objective, selective and arranging the subject matter and determing the method and procedure."

According to **Joseph Landon,** "Lesson-plan as a draft of the lesson put upon paper with all the important points whether of matter, or method clearly marked."

According to **Educational Dictionary,** "Lesson-plan is a teaching out line of the important points of a lesson arranged in the order in which they are to be presented, it may inciude objectives, points to be made, questions to be asked, references to materials, assignment etc.

According to **N.L. Bossing,** "Lesson-plan is the title given to a statement of the objective to be realized and specific means by which these are attained or a result of activities engaged during the period."

According to **Laster B. Stands,** "A lesson plan is actually a plan of action. It therefore, include the working philosophy of the teacher, his knowledge of philosophy, his information about and understanding of pupils, his comprehension of the objectives of education, his knowledge of the material to be taught and his ability to utilize effective methods."

According to **Ryburn,** "To teach we must use experience already gained as starting as starting points of our work."

In this statement **Ryburn** considers that a pupil teacher gains experiences about his classroom. Work through lesson planning, so that he is able to perform his tasks successfully in his teaching.

I.K. Davis has given four steps for management of learning viz. planning, organising, leading and controlling. So, he has given the greater importance to lesson planning as the first step of teaching. He has defined in the following manner:

"Lesson must be prepared for there is nothing so fatal to a teacher's progress as unprepardness."

So, lesson planning is an important and memoryful or intelligent planning of learning, through which we can see that how the childs are motivated for gaining new information, ability and freequency and how it can motivate their previous knowledge.

Need and Importance of Lesson-Plan

Lesson planning plays significant role in the process of teaching. It has its own values and advantages. In teaching-learning process the need and importance of lesson-plan may precisely be mentioned in the following points :

1. Lesson plan confirms the order and shape of presentation of subject matter.
2. Lesson plan creates self-confidence in students and teachers.
3. It provides guidelines to the teacher to proceed systematically in the classroom teaching.
4. Lesson plan establishes relation between teaching activities and learning.
5. Lesson plan makes the teaching work regular and collective.
6. It provides awareness of teaching objectives and structure of content and teacher has to perform his activities in the direction to achieve the objectives.
7. Lesson plan helps the teacher in choosing the best method and the

teaching materials can be made in an order with its help.

8. Lesson-plan creates useful educational environment in the class.
9. The sequence of content is to be presented and finalized by task analysis in lesson-planning.
10. The teacher has the best possible use of time of his class teaching by lesson-plan.
11. Lesson plan inspires the teacher to do proper and important questions.
12. Lesson-plan determines definite teaching objectives of daily lesson.
13. It makes the teacher work regular, well arranged according to plan.
14. Lesson plan is in written form and it enforces the instructional aids.
15. Lesson plan is helpful to distributing the teaching material in to units and to reorganise again it, given in the books.
16. Lesson-plan establishes relationship between different subject materials.
17. Lesson-plan develops logic, thoughts and the power of decision and imagination.
18. It helps the teacher to know and solve about different types of problems and difficulties in connection with the subject matter.
19. Lesson-plan gives attention towards different matters of personality of the child means it may be conducted according to individual teaching work by using individual differences in the lesson-plan.
20. The basis of previous knowledge to achieve new knowledge may be the secret of the success of the teaching.
21. The necessity of lesson-plan is also important in training programme of the teachers. Lesson-plan present the outline for class-activities for training of the teachers.
22. It is observed several times that if the teacher comes in the class without teaching preparation and does not perform the teaching work properly indiscipline is created in the class. The lesson-plan is prepared to avoid from this also. It is helpful to stop the indiscipline.

Principles of Lesson Planning

To make his teaching successful and effective a teacher plans the lesson, prepares it and puts down his plan in the form of lesson note. In this process of lesson planning, the following principles are to be followed:

(i) Selection of suitable subject matter.

(ii) Presentation of the selected material in an organized, orderly and effective manner.

(iii) Child activity and participation in co-sharing the teaching-learning process.

(iv) Attainment of objectives and outcomes of orderly procedure.

Evaluation of a Good Lesson Plan

The following are main characteristics of a good lesson plan :

1. A good lesson plan should preferably be written. It should not be at oral or mental stage because many of details are lost when the plan is put to practice, if it is merely a mental sketch,
2. The psychological units of planning should be well arranged. The plan should contain all the steps and stages of an orderly procedure.
3. The plan must have its basis on the background of the class.
4. The subject-matter and teaching aids should be well selected to suit the pupils.
5. Application stage or the expression work to be assigned must find a suitable place in the plan. References, if there are, should be clearly given.
6. Important and essential elements of the subject-matter should be written at the end of the plan as black-board summary to be reproduced.
7. A good lesson planning requires that the lesson unit must be finished within the specified time. Therefore, time must be apportioned suitably to all stages of the plan.
8. A good lesson plan should be flexible and elastic. It should not be too rigid and mechanical.

Main Forms of Lesson Planning

There are various forms of lesson-plans. The paradigm of lesson-plans differs from country to country. The American approach, British approach and Indian approach are generally applied in developing the lesson plans.

In American approach priority is given to learning objective, so the activities of teacher and students are organised in such a manner that the optimum realization of objectives may be done. The teacher-activities are set to generate appropriate learning situations for bringing the desirable change in learner's behaviour.

In British approach, the emphasis is given on the teacher's activities and presentation of content in the lesson planning. The teacher has to play major role in teaching-learning process. He has to plan, organise and control the student-activities.

The Indian approach of lesson planning has the influence of both the approaches, American and British. So, the learning objectives, teacher-activities, student-activities and evaluation of student are crucial aspects of Indian approach to lesson planning. The Regional College of Education

Mysore has developed a paradigm of lesson plan. It consists of teaching objectives and learning experiences. The question-answer strategy is followed in developing the lesson-plan because teaching is considered as an interactive process. It is commonly known as RCEM approach to lesson planning.

There are various forms of written lesson plan in our country and abroad but following three forms are most popular and commonly used.

1. Herbartian approach to lesson planning.
2. Bloom's approach to lesson planning.
3. RCEM approach to lesson planning.

Herbartian Approach to Lesson Planning

The Herbartian approach is based on apperceptive mass theory of learning. The proposition of that theory is that the learner is like a clean slate and all the knowledge is given from outside. Herbartian approach is also known as Herbartian five steps approach. In most of teacher's training institutions the Herbartian five steps approach of lesson planning is used.

The lesson planning is an ancient concept. So, in earlier periods too, attempts have been made to formulate a general procedure for the conduct of various types of lessons. But the credit of significant contribution in this field is to Herbart.

John Fredrik Herbart, a German philosopher and great educationist and his followers adopted and evolved the most famous procedure known as the **"Herbartian Formal Steps."**

Steps of Herbart's Lesson Plan

J.F. Herbart gave the five steps of lesson-planning. Due to which it is called five-step process of Herbart. Which are as follows:

1. Preparation
2. Presentation
3. Comparision and Association
4. Generalization
5. Application and Repetition

1. Preparation–In this, teacher do preparation for giving the new knowledge. For this teacher necessarily knows that how much knowledge a student has about the topic which is taught in the class. For this he also asks the oral questions to the students. Due to which he knows that what and how a student knows about the topics.

2. Presentation–Teacher gaves presentation after attaining the previous knowledge of topic. He gives presentation with the help of developing questions. Teacher should use demonstration, practical and

other teaching aids for presentation of subject matter to make his lesson plan effective. In this teachers use black-board time to time.

3. Comparision and Association–In this whatever knowledge a student gained, should be checked means how much knowledge a student gained, be checked. For this teacher established relationship between previous knowledge facts of subject matter and new facts of subject matter and he compare the new knowledge of student to the previous knowledge.

4. Generalization–In this step some result must come after teaching of any lesson. Some general rule is prepared according to these results. This process is known as generalization. Generalization in science, maths, grammar etc. is more easy in comparison to other subjects. By this student gets the chance to think deeply and his imagination power is also developed.

5. Application and Repetation–In this step, the knowledge given to students, get chance to make it permanent. The knowledge may become permanent by using the learnt knowledge into new circumstances. The students are given facilities of use of learnt knowledge to prove truth of theories and rules to clarify the knowledge. The teachers come to know through repetition what and how much the student has learnt.

Merits of Herbart Approach

Merits of Herbart process are as follows :

(i) Approach of Herbart is based on psychological principles.
(ii) This approach is based on previous knowledge.
(iii) The form of every steps is fixed and arranged in this approach.
(iv) Herbart process can be used easily in every topics of every subject.
(v) Teacher has an important place in the process of Herbart.
(vi) Knowledge is found in the combined form by the help of this process.
(vii) Students always active in this process.
(viii) By the help of this process previous knowledge checked or trained on the basis of gained knowledge.

Demerits of Herbart Approach

Some demerits are found in the Herbart process which are as follows :

(i) To pay attention towards the presentation of subject matter is the main objective of Herbart process.
(ii) When teacher follows this approach then his freedom becomes limited.
(iii) Teaching of only congnitive domain of education is possible by this process.
(iv) There is the shortage of flexibility in this approach.

(v) It doesn't pay attention towards interest of the students.
(vi) In this teacher is more active in comparison to students.
(vii) It emphasises the way of teaching.
(viii) Teaching of only memory level is possible by this process.

Criteria of a Good Lesson Plan

(i) Lesson planning should be constructed carefully on the basis of subject-matter.
(ii) Language of lesson plan should be clear and easy.
(iii) General and specific objectives related to lesson should be written carefully in lesson planning.
(iv) Lesson plan should be broad in the point of view of teaching method, techniques, objectives etc.
(v) Lesson plan should be written by blue-black ink.
(vi) Lesson plan should be child centred.
(vii) Base of lesson plan should be connected with previous knowledge to create the interest in the students.
(viii) Black-board entries should be filled in attractive way.
(ix) Question method should be effective.
(x) Lesson plan should be based on active cooperation of students.
(xi) A good lesson plan should pay attention towards individual attention.
(xii) Vague and irelevant material should be avoided.
(xiii) It should be in written form, in good hand writing and bold letters.
(ixv) Selection and organisation of the subject matter should be to the point and systematic.

Limitation of Lesson-Plan

Lesson plan has following limitations :

(i) Lesson plan is failed in attaining of specific objective.
(ii) Many times its important points can not be identified properly.
(iii) To fill the entries or work of black-board in incomplete or unattractive way.
(iv) To expect need, ability, age, interest of students in lesson-plan.
(v) No broadness of lesson plan.
(vi) No usefulness of homework.
(vii) Incomplete knowledge in selection of teaching method and do its selection.
(viii) Shortage of co-relation in different topics.
(ix) There is lack of flexibility in lesson planning.

9.3.3 Bloom's Evaluation-based Approach to Lesson Planning

Bloom's approach is based upon the interrelationship among objectives, learning experiences and evaluation techniques. The approach involves the following steps.

– Formulating educational objectives

– Creating learning experiences

– Evaluating the changes of behaviour

Formulating Objectives

In this approach to lesson planning the first step is to formulate objectives and state them in clear and specific behavioural terms. This requires the teacher to assess the entry behaviour of the students so that he/ she will be able to lead them towards the terminal behaviours which are otherwise the learning outcomes.

Creating Learning Experiences

After selecting and stating the specific outcomes of learning, the teacher has to plan and provide the most appropriate learning experiences that would result in the desirable changes in behaviour. This is the step where the teaching and learning takes place. These learning experiences may be varied and different depending on the nature of the content, the learner and the type of learning outcome. This type of lesson planning forces the teacher to consider a variety of activities that are meaningful and goal-oriented. Such activities ensure greater student involvement in learning at every step and keep the students active and alert.

Evaluating the Changes of Behaviour

At this step the teacher selects suitable evaluation tools and techniques to assess the changes in behaviour. Thus evaluation becomes an integral part of teaching. As and when a learning experience is provided, its effectiveness in attaining the objective is immediately evaluated. If the outcome is not favourable, the teacher can plan an alternative learning experience to achieve the objective.

The format of a lesson based on this approach can be as follows.

Objectives	*Learning Experiences*	*Evaluations*

However, content is not given as a separate part, and other steps are not specified. But learning experiences are always based on the content and other steps can be easily incorporated.

9.3.4 RCEM Approach to Lesson Plan

RECM approach is advocated by the Regional College of Education, Mysore and hence the name. The rationale behind this approach is a system approach in education. A system approach demands the presentation of information in a systematic manner.

There are three stages or aspects in RCEM approach - They are:

– The input, otherwise known as Expected Behaviour Outcomes (EBOS)

– Process, otherwise known as communication strategy and

– Output, otherwise known as Real Learning Outcomes (RLOS)

Input (EBOS)

The first stage in RCEM approach is concerned with the identification of the behavioural objectives pertaining to a particular lesson or content.

Process

This aspect of the lesson planning involves presentation and integration of knowledge and skills. The emphasis is on how well the knowledge and skills are communicated to the learners

The Output or RLOS

The output indicates the terminal behaviour or change in behaviour of the students after learning. The output stage in lesson planning represents the evaluative phase in the lesson.

9.3.2 Unit Approach (Morrison's Approach)

The Unit Approach of lesson planning is propounded by Prof. Morrison (1871-1945). This approach emphasises the mastery over the subject matter by the students. The subject matter or the lesson chapter is split into meaningful small segments known as units. The unit approach of lesson planning involves the following steps.

Exploration

The teacher explores various methods and possibilities to motivate the students, to arouse the curiosity and to maintain the interest of the students. This is the preparation step where the teacher could plan for the success of the lesson.

Presentation

This step is basically common to all the approaches of lesson planning which involves the selection and use of different methods to present the subject matter to the students.

Assimilation

The third step involves intensive learning, and deep understanding of the subject matter that facilitate effective communication of the concepts.

Organisation

This is an important step in unit approach to lesson planning as it determines the extent to which students are able to reproduce the material of the unit in writing without any external help. The ability of the teacher to enable his students to reproduce the knowledge reflects the efficiency of the teacher.

Recitation

Recitation in unit lesson planning means that an individual student is able to reproduce the same text orally on the completion of the lesson by a teacher.

Prerequisites for a Good Lesson Plan in Mathematics

However, all the approaches discussed above are tentative guidelines and need not be followed rigidly. All the above approaches emphasise the following as prerequisites for any lesson plan.

- Clearly and precisely stated objectives
- An interesting motivation to introduce the lesson
- Systematic and orderly presentation of the subject matter
- A suitable method with relevant learning activities, adequate teaching aids, examples etc
- Application of the new knowledge to new and daily life situations and integrating it with previous knowledge
- Reviewing and consolidating the learning points
- Evaluating the attainment of the objectives using appropriate evaluation devices.

Steps in Writing a Lesson Plan in Mathematics

For lesson plans in mathematics the following steps can be followed

- Introduction or Motivation
- Announcement of Aim
- Presentation
- Application
- Supervised Study
- Recapitulation and
- Assignment

Components of a Lesson Plan

A successful lesson plan addresses and integrates these 3 Components.

- Objectives for student learning
- Teaching Learning Activities
- Strategies to check student's understanding

Functions and Purpose of Lesson Planning

1. Lesson Planning gives the teacher greater assurance and greater freedom in teaching. The teacher who has planned his lesson wisely ensures the classroom without any worry.
2. It stimulates the teacher to introduce pivotal questions and illustrations.
3. It enables the teacher to know the most desirable type of teaching procedures and to prepare tests of progress and checks for judging the outcomes of instructions.
4. Since lesson Planning establishes proper connections between different lessons or units of study, it provides and encourages continuity teaching process.
5. It provides adequate lesson summaries, ensures a definite assignment for class and availability of materials for lessons when needed.
6. Lesson Planning prevents waste because it helps the teacher to be systematic and orderly, It saves him from hazard teaching.

Some other important functions and importance of Lesson Planning are:

- It provides a coherent framework for smooth efficient teaching.
- It helps the teacher to be more organized.
- Lesson Plan gives a sense of direction in relation to the syllabus.
- It provides a useful basis for future planning.
- It helps the teacher to plan lessons that cater to different students.
- It helps the teacher to be more confident when delivering the lesson.

What are the important aspects of lesson planning for students?

The lesson plan is a guideline through the help of which a teacher can teach well and reach his desired goal. Making use of these guidelines the teacher can deliver the good materials to students in a logical se-quence.

In the absence of a lesson plan the progress of the class is hampered For good teaching we have to use a good method of teach-ing.

Thus the only best way of good teaching is possible is by making use of integrated teaching of content and methods. The teaching of contents through better methods is definitely superior to teaching con-tents. Because of these advantages planning of lessons is considered essential for becoming a good teacher.

Important aspects of lesson planning

Following question should and should be considered before a les-son:

(a) For whole the lesson is planned?
(b) What is the important topic to be taught?
(c) By whom is the unit going to be taken up?
(d) What type of inter-departmental help is required?
(e) How has the basic knowledge been imported?

Types of Lesson Plan

1. Five-Step Lesson Plan
2. Five-E Lesson Plan
3. Weekly Lesson Plan
4. Unit Plan
5. Inquiry-Based Lesson Plan
6. Micro Lesson Plan
7. Macro Lesson Plan
8. Detailed Lesson Plan
9. Semi-Detailed Lesson Plan

We can classify Lesson Plan in many types

1. **Classification 1**
   - Five-Step Lesson Plan
   - Five-E Lesson Plan
   - Weekly Lesson Plan
   - Unit Plan
   - Inquiry-Based Lesson Plan

2. **Classification 2**
   - Micro Lesson Plan
   - Macro Lesson Plan

3. **Classification 3**
   - Detailed Lesson Plan
   - Semi-Detailed Lesson Plan

Format of a lesson plan

Name of Teacher :

Class : Section:

Strength of the class :

Subject :

Topic :

Duration :

Date :

General Objectives

– Knowledge

– Understanding

– Application

– Skill

Specific Objective :

Knowledge :

Understanding :

Application :

Skill :

Previous knowledge :

Aids Used :

References :

Steps / Content	*Specifications*	*Learning Activities*	*Evaluation*
...ent of the Aim Presentation Application Supervised Study Recapitulation Assignment			

Some sample lesson plans are given on some selected topics from algebra, geometry and mensuration.

In brief a lesson plan is a pole star guiding the ship of classroom teaching to reach the final goal.

UNIT PLAN

In modern era, unit plan is used for the removal of defects of lesson planning. For the complication of each unit we plan the different topics. In which each lesson is a part of unit and which is helpful in the development of next lesson. In general meaning unit plan is that planning in which teacher should distribute the whole work related to curriculum in a small units, but these units are significant, by which teacher work concentrately.

The unit plan approach has been devised by **Dr. H.C. Morrison**. He emphasised that children should be taught in such a way so as to enable them to gain a mastery of the learning material. Unit is not a mixture of lesson plans but it is a compound form of them. It is a big Gestalt of a teaching learning situation which can be broken up into smaller constituents not less than two and not more than seven in number which are more popularly called the lesson plan. Each lesson presents a certain fact of the unit. Unit plan permits the application of Gestalt psychology in that an overview of the unit may be presented by the teacher with the help of the pupils before the actual assimilation activity of the unit begins.

Definition of Unit Plan :

According to **Vibasi,** "Unit is the sum total of the activities and learning material organized around some central problem or purpose. It is developed by a group of pupils under the leadership of the teacher."

According to **Preston,** "A unit is related to the student contents, which is a part of in a particular or definite time."

According to **Carter V. Good,** "Unit may be defined as an organisation of various activities, experiences and types of learning around central problem or purpose, developed co-operatively by a group of pupils under teacher leadership."

According to **Morrison,** "A unit consists of comprehensive series of related and meaningful activities so developed as to achieve pupil purposes, provide significant educational experiences and result in appropriate behavioural changes."

According to **Bossing,** "A unit consists of a comprehensive series of related and meaningful activities so as to achieve pupil's purpose, provide

significant educational experiences and results in appropriate behavioural changes."

According to **Heidgerken,** "The most important aspect of the unit concept is the implication that it is larger and more involved than a few scattered facts."

On the basis of above definitions, it is clear that unit plan is a form of learning or experience, which is given by teaching activities. A unit is related to the student contents which is a part of in a particular time.

Dr. H.C. Morrison gives his formula for such a mastery is pre-test, teach, test the results, adopt procedures, teach and test again to the point to actual learning. He formulated five steps of his approach :

(i) Exploration
(ii) Presentation
(iii) Assimilation
(iv) Organisation
(v) Recitation

Need and Importance of Unit Plan

The main need and importance of unit plan is :

1. A unit includes various types of teaching activities.
2. A unit provides an opportunity for teacher and pupil interaction.
3. It provides the basis for evaluating pupils performance.
4. A unit is to consist of purposeful related activities which should give insight into and increase control of some significant aspect of environment.
5. A unit initiates new activities which are not possible during the class period.
6. It extends student experiences beyound the limits prescribed by the syllabus.
7. It anticipates future needs e.g. illustrative material to be borrowed, developed.

Purpose or Objectives of Unit Planning

"A multiplicity of material is almost a necessity for teaching process if the class is to realise the objectives." —***Hanson***

On the basis of this unit plan has following objectives :

(i) A unit gives more emphasis to the student's learning.
(ii) Organisation of content is according to the need and interest of the students.
(iii) Give more importance of home assignment and drill work in science or maths.

(iv) By the help of this we divide the content in different sub-units on the basis of problems, theories and laws.

(v) Give a systematic knowledge to principles, theories and concept etc. in the maths.

(vi) Determination of desired methods and techniques for evaluation.

Characteristics of Unit Planning

"A unit plan leaves pupils free to work. They have chance to plan, organise and execute." —*Schorting*

There are many characteristics of unit plan, some of them are of follows :

(i) The material of the unit plan should consists of familiar and related topic.

(ii) It is psychologically sound approach for effective teaching learning process.

(iii) It develops the qualities as self-confidence, persistance, security etc. in the students.

(iv) Teacher and student both are coporative in the preparation of a unit.

(v) It provides opportunity to the teacher to do experiments with his over ideas in-colaboration with pupils.

(vi) It is flexible to deal with a wide range of learning-situation.

(vii) The length of unit is maintained the interests of the students.

(viii) The previous knowledge, experience and background of the student is to give due importance.

(ix) It facilitates the use of a wide range of resources such as laboratory equipments, charts, films, as an integral part of the learning experiences.

Types of Unit Plan

According to **James Michael**, unit plans are two types :

1. Teaching Unit
2. Resource Unit

1. Teaching Unit–A teaching unit concerns with subject-matter or content and teaching strategies. The teaching practice can be organized with the help of teaching units, these units related teaching to learning. There are three steps in a teaching unit:

- Introductory phases of lesson-plan.
- Interactive phase of teaching.
- Situation to measure the students performances.

2. Resource Unit–A resource unit is one which is intended to be a general guide in assistence the teacher to enrich the teaching unit. Thus,

the resource unit is more general and comprehensive than the teaching unit.

Steps of Developing Unit Plan

Following steps are followed to developing the unit plan :

1. Preparation
2. Previous Knowledge Test
3. Presentation
4. Summarization
5. Drill
6. Evaluation

1. Preparation–Firstly motivate the students before the teaching. For the purpose teacher should prepare plan.

2. Previous Knowledge Test–Teacher should check the previous knowledge of the students. Student's previous knowledge is tested in such a way that curiosity may arouse for learning something new in the mind of the child. This should be done by linking their previous knowledge with the new learning material.

3. Presentation–In this step the methods and techniques employed are related to the subject matter. Material is presented to the students in an orderly manner with suitable examples, taking in account the understanding the power of child, proper questions-answer technique is employed to develop the subject matter with mutual participation of the teacher and taught. Proper illustration and aids are used according to the needs.

4. Summarization–Teacher should present the summary of the lesson or content after the completion of the chapter or lesson. For this purpose teacher tell the content related matter effectively.

5. Drill–In mathematics drill work is very important. He present the lesson in a interesting way for continuous drill and the knowledge can be permanent.

6. Evaluation–Evaluation is a important and last part of unit plan. By this help of this teacher relate the desirable behaviour change in the student. For the evaluation purpose teacher can be use the questionnaire or test.

Limitation of Unit Plan

Following Limitations of unit plan are :

1. Unit planning is not aim of goal.
2. In this too much of plan-centred.

3. In this too much time and labour demanding.
4. Planning of unit plan is not proper in work.
5. Evaluation work is not possible in small classes.
6. For this purpose we need the experienced, trained and dedicated teachers.
7. The planning is not to develop the logical-power, memory-power and observational power etc. in the students.
8. Teaching becomes mechanical.

Principles Involved in Unit Planning

1. Principles of Unit
2. Principles of Interest
3. Principle of Development
4. Principle of Dynamism
5. Principle of Organisation

Level of Units

They are three type :

1. Yearly Planning
2. Monthly Planning
3. Weekly Planning

Format of Unit Plan

1. Class ..... subject ..... Name of Unit .....
2. S.No. of Unit (Yearly Planning) .....
3. No. of periods required for the unit .....
4. Duration of the period .....
5. No. of periods for Recapitulization .....
6. No. of periods for evaluation .....
7. No. of periods for remedial teaching .....

Sub. unit and topic	Teaching points	Specific Objectives & Expectual change in behaviour	Teaching Learning Situation			Home Assignment	Evaluation
			Teacher's activities	Student's activities	Audio-Visual aid		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

❑❑❑

Advantages of Unit Plan

Unit planning contributes to the educational process in the following ways.

- Unit plan beaks up a lengthy unit into smaller sub-units or topics so that pupils can easily grasp the scope of these during a brief overview.
- It helps the teacher to present the various principles and concepts constituting the unit in an orderly and systematic manner, without losing their continuity.
- It enables the pupils to see clearly the relationship between the various facts, processes and principles that make up the unit.
- It helps the pupils to appreciate the unifying principles linking all the information together in the unit. This guides pupils to view the sub-units as part of a whole and not independent segments of information.
- It helps the teacher to plan a variety of learning experiences, keeping in mind the individual differences, the nature of content and objectives to be achieved.
- It provides frequent opportunities for the students to review and reorganise their learning.
- It helps the teacher to plan definite outcomes of learning so that they are clear not only to the teacher, but also to the students.
- The study outline of the unit plan provides the students with directions as to what to study, and how to do it most effectiviely.

9.1 Year Plan

Year planning is done by the teacher at the beginning of the year for the entire course. For example, a teacher who teaches a course in mathematics for a particular class plans the curricular and co-curricular activities as per the syllabus for the entire academic year. The teacher has to prepare the year plan keeping in mind the following points.

- The number of units to be covered as per the syllabus during the academic year.
- The number of periods needed to cover each unit, the number of periods allotted for mathematics per week.
- The number of working days per term and for the year.
- The number of days allotted for project work, revision tests and examinations.
- The number of holidays during the academic year.

The year plan forces the teacher to consider the time available and make the optimum use of it, by planning per week, per month and per term before the commencement of academic session. The year planning helps the teacher in taking decisions regarding when to teach what, how much time can be assigned for each unit / topic, how many hours of project or laboratory work can be assigned to the students, how much time can be allotted for revision, tests and examinations etc. Therefore, a year plan ensures smooth instruction through- out the year and helps the teacher to carry out all the curricular and co-curricular activities in time without haste and anxiety.

Advantages of Year Plan

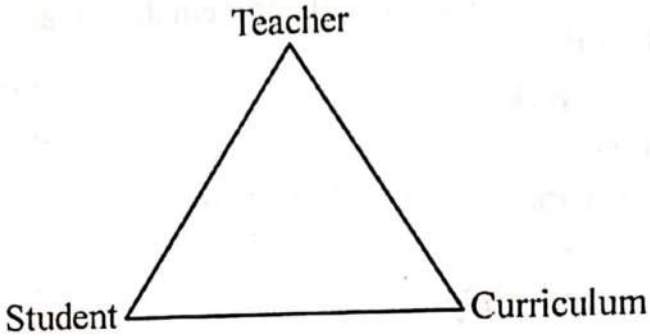
- Year planning helps the teacher in planning for the entire course for the whole of the academic year.
- It keeps the teacher on the right track.
- It enables the teacher to cover the syllabus within the allotted time, as planning saves waste of time and energy.
- It helps the teacher in orderly and systematic teaching
- It gives a lot a self-confidence to the teacher as she is sure of what is expected of her during each period /week/month/ term etc.

TEACHING METHODS OF MATHEMATICS

Every method has some goodness in it, no method is all good. Children should be told as little as possible and induced to discover as much as possible.

—*Valtaire and Spancer*

'How to Teach' is a really difficult problem for the teacher. It's already been discussed that teaching has three poles—Teacher, Student and Curriculum. These are interrelated to each other. Teaching can not go smoothely in the absence of any one of these three. These three points make a triangle, which is impossible including one of these three.



The word 'method' has been derived from Latin word which means, "Mode" or "Way". Therefore here it means, method of delivering knowledge and transmitting mathematical skills by a teacher to his pupils and their comprehension and application by them in the process of learning mathematics.

Generally, maths is considered to be a difficult subject. So it is the responsibility of the teacher to present it in a way that the student, can understand it easily. One method can not be effective for every student. So the teacher has to use different methods of teaching to achieve goals in the minimum time.

In a very restricted sense, it means "what to teach?" and "How to teach mathematics?" or "How to approach it?" Hence, "The process of interpreting the world to knowledge to pupils mind is called the method of teaching."

The teaching of mathematics also involves different methods. The mathematics is taught at different levels of school education. No one single

method is fully suitable for each level. Thus, different methods are used for the students of different level. Teaching methods are not meant to provide only knowledge to students but to bring virarity to teacher-student relationship. A teacher with the help of teaching methods enhances student's personality, intelligency, emotions, values and character.

There are different methods of teaching mathematics for primary, secondary and higher level education as the students. One method can not be applied to teach mathematics to all the classes. Besides this, with the use of one method, all the students of a class can not be equipped with equal amount of knowledge because individual difference lies among them. Individual difference is an important psychological phenomenon which affects teaching and its outcome to a great extent.

Before selection of teaching method a skilled teacher first decides the objectives of teaching of that particular subject? and How these objectives can be achieved easily? How should the contents be taught? To answer all these questions different methods of teaching of mathematics are adopted for different situations. These teaching methods helps to achieve objectives easily.

By the use of teaching methods a teacher is benifites in these ways:

1. Teaching methods provide right direction and pace of teaching.
2. They are directly related to teaching objectives, so they help to achieve them easily.
3. They emphasis on 'How' which satisfy students curiosity.
4. They lay stress or not only reasoning and psychological aspects but social, philosophical and economic concepts etc.

Definitions of Teaching Methods

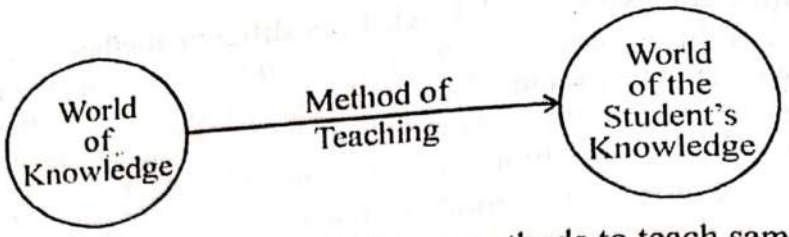
According to **Talleyrand,** "Method is the master's master".

According to this definition a teacher teaches only lesson, but these methods provide a structure of the lesson by the help of these methods teacher teaches effectively the lesson to the students.

According to **Kothari Commission,** "If mathematics is done bodily, it is worse than unless".

According to **George W. Hunter,** "A teacher may be of great value, may lose much of its value in the hands of another teacher, especially, if that teacher believes that a different method is better."

On the basis of above definitions, we can say that no two persons are similar to each other then why teachers teach them in same method. The world of knowledge includes, the knowledge, interest, attitude, skill etc. i.e. all the three domains-cognitive, affective and psychomotor. It is explained by this figure :



So, a teacher has to apply different methods to teach same topic to same class so that every student could understand and learn the given content.

Precaution for Selecting Teaching Methods

A teacher's work is not only to teach students with the help of teaching methods but to provide them more and more knowledge by selecting the most suitable method. A skilled teacher should take the following measures while selecting the correct or suitable method :

1. Teaching method should develop various skills, independancy, self-confidence among the students.
2. Such a teaching method should be selected by which the knowledge given be purposeful, concrete and real.
3. Such method should be selected by which the children get proper opportunities to apply or use the acquired knowledge.
4. The children should be given opportunities to work together in groups.
5. Teaching method emphasis should not be given on verbalism and memorization.
6. The teaching method should develop different skills, honesty, self-dependence and self-reliance amongst the children.

A successful and experienced teacher should keep in mind, the mental level, interest, habits and attitude of children before selecting a method. Along with this the teacher should also keep in mind the individual differences, because by using proper and effective method, the children take interest in mathematics.

Need of Teaching Method of Mathematics

"There is lot of difference between high I.Q. students and low I.Q. students in the same class. The high I.Q. Students learn six times more of the low I.Q. students, in other words, low I.Q. students learn only sixth part in comparision of high I.Q. students." —***Thorndike***

This definition indicate that high and low I.Q. students are found in class, which are not taught in a same teaching method. By the help of these methods desirable changes occur in the students.

Man is a social animal. He born and developed in a society. He learns

in his environment as well as school. One method can not be applied to teach mathematics to all the classes. Besides this, with use of method all the students of a class cannot be equipped with equal amount of knowledge because individual difference lies among them. Individual differences is an important psychological phenomenon which affects teaching and its outcome to a great

Importance of Teaching Methods for Mathematics

Teaching method play a important role in a mathematics teaching. It is shown by following facts—

1. Typical facts can be understood by play method in a small classes.
2. Teaching method is created the freedom and self-discipline in the students.
3. By the analytical method student always motivates to achieve new knowledge.
4. Synthetic method is a unpsychological method, which built the memory level strongly.
5. Inductive method develops the mental ability from specific to normal, from known to unknown, from complex to simple.
6. Deductive method develop the mental ability from normal to specific, unknown to known, simple to complex.
7. Students solve the problem by learning by doing in a analytical method.

Factors Affecting the Teaching Method

As it has already been mentioned that teaching process depends upon teacher, student and curriculum and its first objective is to achieve instructional objectives and second objective is learning experience and third is evaluation. A good teacher uses many methods to make his teaching effective but these methods too are affected by many factors–

1. Factors Related to Students–Child is the centre of any teaching methods so this mental level, ability, interest will to learn and attitude affects the selection of teaching method. Therefore, the teacher should keep all these things in his mind before selecting any specific method.

2. Factors Related to Teacher–If student is the one point of teaching process then the other is the teacher. The teacher has most active role in teaching process. The teaching process can be made more powerful and effective by using teaching methods. Hence, teachers choose or select the method of teaching according to student and subject matter.

3. Factors Related with Content–Content is the base of communication between students and teacher and it is the base of teaching also. A specific nature of content directly affects the selection of the

teaching method. So the school curriculum should be based on the principles of "Activity centred" or "Learning by doing" and "Learning through Experiences". By this learning child become more effective.

Difference between Teaching Methods and Techniques

According to **I.K.Davis,** "Strategies are broad methods of teaching and instruction".

Teaching Method	Teaching Techniques
1. Teaching methods are based on classical theory of human organisation.	1. Teaching techniques are based on modern theory of human organisation.
2. Teaching methods have direct relationship with aims of teaching.	2. Teaching techniques have indirectly related with aim of teaching.
3. Direction and speed of teaching is decided.	3. It depends upon the main method of teaching.
4. It is emphasis on 'How'?	4. It is emphasis on 'with', 'what' or 'when'?
5. In teaching method macro approaches are followed.	5. In teaching strategy micro approach is followed.
6. The criteria for evaluating the teaching method is the mastery on contents.	6. The norm of evaluation is the acquisition of objectives.
7. In teaching method, the task and its presentation are considered important.	7. In teaching techniques, the behaviours and relationship are considered more important.

Teaching Methods of Mathematics

Different methods of teaching have been proposed or propounded by different educational thinkers or schools of thought in education. It is but desirable for the student to know about all of them, so that he can make a rational choice for himself. The knowledge of procedures, merits and demerits of all the methods will broaden the outlook of a would-be teacher. The choice for him is not to be made narrow. It should be then left for him to decide from his work information, which of the methods is to use and when.

The following methods have been discussed in detail—

1. Lecture Method
2. Demonstration Method
3. Lecture-Demonstration Method
4. Analytical Method
5. Synthetic Method

6. Inductive Method
7. Deductive Method
8. Heuristic Method
9. Laboratory Method
10. Project Method
11. Assignment Method

1. LECTURE METHOD

Lecture method is a very common method of teaching. This is a teacher centred method. This method plays a very important role in classroom teaching. A teacher can achieve a lot through by this method. One of the reasons is that the teacher can teach easily without the practical work. In other words in this method the teacher is an active participant and the child is a passive learner. Lecture means, the methodological presentation of an idea through speech. In this method teacher delivers a lecture on a particular topic and the children listen. It is very attractive method because in it teacher remains the centre of the teaching-learning process. But now-a-days it is not considered effective because it is not useful in gaining knowledge and changing of behaviour.

According to **Carter Good,** "An instructional procedure by which the lecture socks to create interest to influence stimulate or moved opinion, to promote activity, to impart information or to develop critical thinking, largely by the use of the verbal message with a minimum of class participation, illustration, maps, charts or other visual aids may be employed to supplement the oral technique".

On the basis of above definition lecture method has four basic aims:

(i) to motivate
(ii) to clear
(iii) to review
(iv) to expand

When the teacher uses short or lengthy explanations to clear some points of the subject is known as lecture method. Teacher can shorten or lengthen his explanation according to situation. Which saves time and more and more students can acquire knowledge in the least time But in lower classes neither students nor teacher is benefitted.

Many mathematics call that lecture method is a one channel method, because only teacher is active in this method. It does not establish communication between teacher and student and makes the class boring.

It neither gives opportunities to student to response and nor inspires them. Some times students have to listen even when they are not interested at all. So they become bored and sometimes yawn and even work is when they sleep.

Not giving attention to the objectives of cognitive and affective domain this method has become effectless.

For teaching by lecture method, the teacher prepares his talk or theme of the lecture before teaching and pours it out in the class. During this process the students sit silently, listen attentively and try to catch the point. Undoubtedly the process of this method indicates that it is not a suitable method to teach mathematics specially for lower classes.

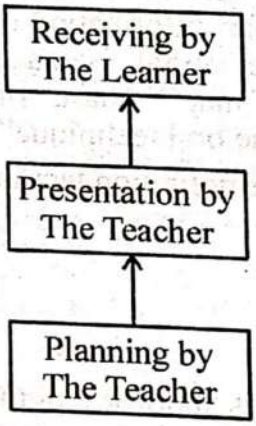
Steps in Lecture Method

There are three steps in the process of lecture method. That is

1. Planning by the teacher,
2. Presentation by the teacher,
3. Receiving by the learner.

It is clear that there is no place of pupil's activities. The teacher is active only and the pupil remains a passive learner. He listens the lecture of the teacher.

It is a convenient method for the teacher because teacher gives more information in a short period. The flow of thought is maintained and the teacher tells about many new things.



Precaution Using Lecture Method

The following precautions should be taken while using this method :

1. Teacher should be pre-decided the duration of lecture.
2. The teacher should have a complete knowledge about the subject.
3. The teacher should be taken care to the main points.
4. The teacher should not use notes or book at the lecture time.
5. Lecture should be according to mental level and interest of the students.

6. The language of teacher should be easy, clear, interesting and full of illustrations.
7. There should be an interaction between the teacher and the students.
8. Suitable teaching aid should be used while giving lecture.
9. Lecture should be short.
10. Lecture should be informal.

Merits of Lecture Method

Lecture method has the following merits :

1. This method is convenient and interesting.
2. When the number of students in a class is very large, this method is the only way out. The teacher's voice is heard clearly even in the farthest corner of the class-room. All the students are provided with an equal opportunity to listen and learn.
3. This method is convenient for the teacher, because he does not have to give individual help.
4. It is economical because it doesn't need practical, demonstration and any equipment.
5. By this method, big or large topics can also covered in very short time.
6. It can accompany many other methods.
7. More information may be given in a short time.
8. It can establish logical sequence of the subject as the lesson is already preplanned.
9. It is useful in starting new topics or knowledge and revising of the old topics or knowledge quickly.
10. It is useful in giving knowledge of the new topics and in explaining them.
11. This method gives a sense of satisfaction to both, the teacher and listener. Teacher feels satisfied that he has finished a certain part of syllabus successfully and students feel that they have learnt something.
12. By means of this method every student gets an equal opportunity to listen and learn.
13. It is helpful in maintaining the discipline. If a lecture is delivered impressively students listen it attentively with pin drop silence.
14. Certain topics like insurance, budget, interest and income-tax, an introductory talk may be found most impressive and useful.
15. It is an inspiring and motivative method.

Demerits of Lecture Method

Inspite of having the above mentioned merits this method has the following demerits :

1. In this method students remain only a passive listener.
2. It is a teacher centred method.
3. It does not develop the scientific attitude in the students.
4. It is oral and verbal method which is not suitable for lower classes.
5. This method gives a false sense of satisfaction, which is dangerous and harmful.
6. This method ignores basic principle of learning by doing.
7. In this method experimentation is totally neglected. There is no opportunity for the students to discover and find out facts for themselves.
8. Home work is likely to be very heavy.
9. It does not impart efficiency in problem solving.
10. It does not develop student's reasoning power.
11. It is dominating method which is against democratic feelings.
12. It is not effective for lower classes as the students are not mentally developed at this level.
13. In this method there is no possibility of interpersonal contacts between teacher and students. Therefore, teacher can not know the difficulties and potentialities of student and so there is no scope for individual attention and guidance.
14. In this method bits of information is received and momorised by the students, so it is not suitable for teaching of mathematics.
15. There is no consideration of the intelligence, abilities and interests of the students.

Conclusion

From the above discussion, it is clear that lecture method is an informational method which goes against independent and original thinking of the learner. This method is not suitable for teaching of mathematics because there is no participation of child in the learning process. Mathematics cannot be taught effectively without participation of students, so it is defective and unsuitable method.

2. DEMONSTRATION METHOD

Demonstration means 'to show'. Demonstration is done in many ways like-Demonstration skills, addition, subtraction, multiplication etc. of different numbers.

The method which encourages verbal interaction between students and teacher is called Demonstration. In other words, in this method both the

teacher and pupil or students are active. The teacher makes a theoretical investigation and proves it in the class-room. Demonstration may be performed by a single teacher or a group of teachers. The teacher should emphasize major points in the demonstration and preferably should write them on black-board.

Precautions for Using Demonstration

Demonstration method is the most effective method of teaching. But its success depends on the way it is used. So the teacher should keep these points in mind while using this method :

1. The thing demonstrated should be explained in simple, easy and interesting language.
2. The teacher should be well-versed in the handling of the apparratus and equipments.
3. The teacher should prepare the out-line the all demonstration after this demonstrate it.
4. The thing for the demonstration should be as big that can be visible to every student of the class.
5. The articles for the demonstration should be kept on the table without an unnecessary article. That could divert the attention of the students.
6. The demonstration should be related to our day life and previous knowledge.
7. All the articles and apparatus should already be arranged in order.
8. Any question raised during the demonstration should be answered and the students should be told to write the experiment and its result in practical note book.

Merits of Demonstration Method

It has the following merits :

1. It make the topic more interesting.
2. It is economical because it is appropriate in the conditions where there are not enough apparatus for the students.
3. The pupil develop the power of observation, reasoning and thinking.
4. It is less time consuming.
5. Demonstration method is appropriate for lower classes.
6. By this method students learn by seeing things in concrete form not imagination so the knowledge acquired is permanent.
7. It is useful where the experiments involve some complex and difficult operations.
8. The pupil can understand the principles/laws or formulae clearly.

Demerits of Demonstration Method

This method has the following demerits :

1. This method does not provide chance to all students to experiment.
2. It is not based on the principle of learning by doing.
3. Teachers are more active in it.
4. General facts, principles and scientific skills are not easily explained through this method.
5. The whole syllabus can not be taught through by this method.
6. It does not develop analytical attitude.
7. Contents can not be finished in time.
8. This method is costly. So can not be adopted in all schools.
9. Some pupils do not observe properly by this method.

Conclusion

In the mathematics teaching demonstration method is very important. The teacher should emphasize major points in the demonstration and preferably should write them on black-board. The teacher should be well-versed in the handling of the apparatus and equipments. While demonstrating teacher should ask some reflective type of questions to stimulate the power of reasoning and interest of students in the class-room.

3. LABORATORY METHOD

It is also called 'Experimental Method'. To make teaching of mathematics meaningful, effective, interesting and clear it is not enough to teach only but providing opportunity of learning by doing, for which an appropriate method is needed. So this method is mainly adopted because is based on, 'learning by doing', 'learning by observation', 'from concrete to abstract', and 'from known to unknown' teaching maxims.

Laboratory method is a more elaborated and practical form of the inductive method. Pupils do not only listen for information, but do something practically also. The knowledge learnt and acquired by practical method is permanent and cannot be forgotten. The laboratory method is helpful to develop in students the attitude of discovery since they arrive at the solution of a problem by means of experiments. This method can easily be applied in a class-room and also out of class-room.

In laboratory method the pupil do experiments and reach to some conclusion. It keep's students all the senses active and they learn easily, so it is a easy and effecitve teaching method.

According to **Young,** "Abstraction and generalisation are rather the crowing products than foundation stones."

This method needs a laboratory in which equipments and other useful teaching aids related to mathematics are available. For example– various figures and shapes made up of wood or hardboard, logarithmic tables, slide rule, books and magazines related to mathematics, prism, cone, cube etc.

Procedure of Laboratory Method

According to **J.W.A. Young,** "The laboratory method aims to arouse teachers to a belief, not only theoretical but practical and effective as well, that mathematical dishes must be made appetizing and palatable. They are to be accepted with pleasure and digested with ease."

There are no general specified steps to be followed in the use of this method. The teacher and students have to proceed according to the need of the problem and experiment.

The following examples illustrate the use of this method.

Problem : To find the formula for the area of a rectangle.

Teaching Aid : Graph paper, pencil, scale, drawing board etc.

Formula Used : The total number of unit squares lying within the figure of the rectangle is the area of the rectangle.

Method or Procedure : Firstly fix a graph paper on the drawing board with the help of board pins. Draw the figure of a rectangle of specific length and breadth. Then count the squares of graph paper lying within the figure of the rectangle. Now draw another rectangle of different length and breadth on graph paper and again count the squares lying within the figure of the rectangle. Write these observations and finding on the observation table. This process should be repeated by drawing four-five rectangles of different lengths and breadths. The findings regarding area of different rectangles should be noted in the observation table as given—

Observation	Management of Rectangles		Total no. of Unit Squares Lying withing the Figure of Rectangle on the Graph Paper	Area of Rectangle
	Breadth	Length		
1.	5 cms.	4 cms.	20 Squares of 1 Squares cm.	20 Squares cms.
2.	8 cms.	4 cms.	32 Squares of 1 Squares cm.	32 Squares cms.
3.	10 cms.	6 cms.	60 Squares of 1 Squares cm.	60 Squares cms.
4.	7 cms.	5 cms.	35 Squares of 1 Squares cm.	35 Squares cms.

Analysis of Observed Data

After preparing observation table, its data should be analysed to reach on certain conclusion. From the above observation table we can easily analyse that in each observation the area of rectangle is equal to the product of its length and breadth.

Conclusion

The area of a rectangle is the product of its length and breadth. Therefore, in the form of a formula it can be written as :

Area of Rectangle = Length × Breadth

Merits of Laboratory Method

It has the following merits :

1. This method is based on the 'learning by doing', 'abstract to concrete' and 'from known to unknown' teaching maxims.
2. Laboratory method is interesting and joyful for the learner. He likes to be something with his two hands.
3. The knowledge acquired by this method is more solid and durable.
4. It develops scientific attitude in the students.
5. It is a natural method of research.
6. It enhances observation and logical ability.
7. It provides opportunities for creative work.
8. Some topics of mathematics are best understood through this method.
9. It provides great scope for independent work and individual development. It helps in the growth of self-reliance.
10. The problems of algebra and geometry can be solved easily by the use of this method.
11. It helps in developing the habit of discovery and self-study.
12. In this method knowledge is acquired through combined use of different sense organs and motor organs, so it is real and permanent.
13. This method helps to develop various good qualities in the learners such as honesty, concentration, observation, innovative thought and hardworking etc.

Demerits of Laboratory Method

The following are some major drawbacks and limitations of this method :

1. It is expensive method, so this method can not be adopted in all schools.
2. It is very time consuming, so the syllabus can not be completed within time.
3. This method is not applicable or fit for all pupils.
4. Every teacher is not able to use this method effectively.
5. This method can be used for a small class only.

6. All the topics of mathematics cannot exclusively be taught by this method.
7. It fails to achieve the objective without well planning and supervision.
8. It is an exceedingly laborious and slow method.
9. It may sometimes degenerate into a kind of manual training only.
10. Sometimes students take practical task as an end and thus, they remain at the concept stage only. Such students do not reach upto abstract stage which is ultimate goal of mathematics.

Conclusion

From the above discussion it can be concluded that inspite of so many advantages this method also has several drawbacks. Undoubtedly this is an interesting and useful method of learning for science subjects but for teaching of mathematics it should be used only as a complementary method to develop interest in the learning of certain mathematical concepts.

Difference between Laboratory and Demonstration Method

The difference between laboratory and demonstration methods are as follows :

Laboratory Method	Demonstration Method
1. By this method creative and searching habits can be developed in students.	1. In this method students do not get opportunity to develop their creative and investigation powers.
2. Scientific attitude developed in students by this method.	2. It provides very less opportunities for the development of scientific attitude.
3. This is an expensive method.	3. This is not an expensive method.
4. This is a psychological method.	4. This method is not appropriate from the psychological point of view.
5. Students are more active in this method in comparison to teacher.	5. In this method teacher is more active than students.
6. This method takes more time.	6. It takes less time.
7. In this method students get direct experience by experimenting individually in the laboratory.	7. In this method teacher himself demonstrates the related experiment while teaching in the class-room.

4. HEURISTIC METHOD

Like other methods Heuristic method also has a unique place in mathematics teaching. This method is also known as 'Research Method' or 'Self Learning Method'. The heuristic method is the process of leading the pupil to find truth for himself by independent thinking. **Henry Edward Armstrong,** the professor of chemistry in Impreial college, London, propounded this method. It's Greek meaning is 'To discover'. Firstly this method was used for mathematic teaching.

The word 'Heuristic' has been derived from Greek word 'Heurisco', which means 'to discover'. The word has a story. When Archemedise distovered his well known principle of weight in liquid he came out and shouted 'Ureka-Ureka' which means 'I have known' or 'I find out for myself' since the method of self learning is called Heuristic Method. So Heuristic method is a method of discovery. Here child or pupil is put in the place of discoverer.

This method is more important from educational point of view because in this method pupils or students work like a researcher and solve the problems. By the help of this method teacher develops the scientific and mathematical attitude in the pupil or student.

Definitions

According to **Armstrong,** "Heuristic Method of teaching are methods, which involve our present students as far as possible in the attitude of discovers, methods which involved their finding out instead of merely told about things."

In short :

"Heuristic Method is a method, in which we treat the student as a discoverer."

According to **Westawey,** "The Heuristic method is intended to provide a training in method, knowledge is a secondary consideration all together."

According to **Herbert Spenser,** "In this method increase the students more for discovery instead of merely told about things."

According to **Pro. Yound,** "The teacher is the task of that text-book, that the work which he has to do and the problems to be solved, he must keep them in such a way that the student may be able to discover in a real sense. Along with this, it may also be considered that the problems may not be out of his approach and in the end he may achieve the better knowledge of that subject."

In this method the teacher introduces a problem infront of the pupil or students and encourages them to solve it themselves. so that they can

learn by doing. Students cooperate one another and observe to solve the problem. Being active students search truth themselves. Pupils are helped by the teacher whenever necessary, so according to nature of student or pupil they gain new knowledge. This method uses the 'Learning by Doing', 'Practice makes a man perfect' teaching maxims. This method is formative not informative.

Place of Teacher in Heuristic Method

Without the help and guidance of teacher a child cannot proceed properly at his own. So, teacher occupies an important place in this method though his role is confined to certain conditions as given below :

1. He/she is only to act as a guide.
2. He/she should not give the pupils the finished product of his/her own thinking.
3. He/she should carefully select the material and offer his help in the form of question, suggestions and somethings direct telling.
4. The teacher should have in mind the objects of this method and help the child only when he has failed to overcome his difficulties by his own efforts.

Steps in Heuristic Method

According to **Hamley,** these are the steps of Heuristic method :

1. Creation or statement of the problem
2. Discovery of facts
3. Formation of hypothesis
4. Testing of Hypothesis
5. Drawing conclusion

1. Creation of statement of the problem–In this step the teacher introduces a problem in such a way that every student of the class will be familiar to it.

2. Discovery of Facts–In it every student gathers problem relating facts and he asks questions on it. The teacher takes help of books, equipments and other things and guide the students if needed so that they can search and collect facts.

3. Formation of Hypothesis–In this step after collecting the facts, the student formulates hypothesis on the basis of his experiences.

4. Testing of Hypothesis–The student discover or experiment to search the facts in favour or against the hypothesis. They test the truthness of the hypothesis on the basis of many facts.

5. Conclusion–In this step, the student leaves these hypothesis, which are found false and select the true hypothesis and reaches the conclusion.

Principles of Heuristic Method

It has the following principles :

1. Principle of Learning by doing–Human mind is not a store house which collect all the facts. On the contrary is a living process which selects the facts and to solve the present problem.

2. Principle of Purposefulness–The students remain active to solve the problem. For that he follows some rules and principles and find some new principles. It develops the mental level of the students.

3. Principle of Freedom–The student is completely free to choose activity, thinking and research to him.

4. Principle of Logical Thinking–Learning process completely depends on logical thinking. So, to teach anything the teacher should use student's logical thinking.

Merits of Heuristic Method

The main characteristics of Heuristic method are as follows :

1. The student becomes an active participant in the learning process.
2. It develops the habit of self-study, hard working, self-dependence etc.
3. This is a psychological method.
4. It develops the power of observation.
5. It develops scientific attitude, so the student does everything logically.
6. The method suits the learners and the subject.
7. The knowledge gained is permanent.
8. The student thinks for himself and does not merely listen for information.
9. There is no burden of homework for the students.
10. Individual attention of the teacher is possible and the relation between the teacher and learner becomes more intimate.

Demerits of Heuristic Method

It has the following demerits :

1. It is difficult to teach the students of lower age through this method.
2. It is not appropriate method to teach all subjects.
3. It is a very slow method.
4. It demands extraordinary labour and special preparation from the teacher, who is already over-burdened.
5. It is a costly method because so many things may be required to discover a new thing.

6. This is formational method rather than informational.
7. The teacher has to give well-measured guidance, neither more nor less. Over guidance may also harm the initiative of the learner and make him dependent.
8. It may not be possible to teach all topics by this method.
9. This method is not suitable for a class having larger number of students because it demands individuals attention to all students by the teacher.
10. Sometimes teacher is unable to frame good and imaginative questions to provoke real thinking of the child, so it fails the purpose of the method.

Conclusion

Heuristic method is a psychological and scientific method. It provides opportunity of independent thinking and active participation of students. In this method teacher should frame the hints and instructions very carefully to avoid over-feeding and under-feeding. He should let the student or pupil be his own teacher, and also see that his difficulties are removed in time. Whatever be this method of teaching, the guiding principle should be the adoption of the heuristic approach.

5. ANALYTIC METHOD

The word 'Analytic' is the objective form of the 'Analysis' which means the resolution of a compound into its component or parts or analytic method is concerned with the process of breaking the complicated problem into such types of simple and small problems which can be solved by the learner. This method is more effective in geometry. With the help of this method we analyse the typical part of the problem. In this method, we proceed from 'unknown to known' or from 'conclusion to hypothesis'.

Analytic method is a logical method. This method is used psychological area in a era time. Today develop this method in a scientific method or way. It is accepted that a new problem can be solved on the basis of previous knowledge, thinking, reasoning and insight. So in analytic method we start to discover the solution of the problem from its whole form. So, in this method, firstly one has to know 'what is to be found out? Obviously it is unknown for the learner. Then think of possibilities that may connect the unknown with the known. Thus we seek the requirement to get the value of unknown. In this process one requirement leads to another requirement and thus we reach to the ultimate requirement which is known in the question.

According to **Thorndike,** "All the highest performance of the mind is analysis."

This method is used in the following conditions :

1. When we have to prove any theorem.
2. When construction work is to be done in the geometry.

Steps of Analytic Method

This method have three steps :

1. Presentation of Problem–Firstly in this method teacher presents the problem in front of student or pupils.

2. Self-actualisation of Problem–Pupils or students listen this problem carefully and understand its real meaning.

3. Analysis of Problem–After understanding the problem students divide the problem into logical parts which are able to give the solution of problem.

According to **Analytic Method (unknown to known)**

$$\frac{ac+2b^2}{bc} = \frac{c^2+2bd}{cd} \text{ are True}$$

If $$\frac{ac+2b^2}{b} = \frac{c^2+2bd}{d}$$

$$d(ac+2b^2) = b(c^2+2bd)$$

$$dac + 2b^2d = bc^2 + 2b^2d$$

$$dac = bc^2$$

$$ad = bc$$

$$\frac{a}{b} = \frac{c}{d} \text{ which are true.}$$

Merits of Analytic Method

The main characteristics or merits of analytic method are as follows :

1. This method develop the logical, thinking, decision making power etc. in the students.
2. This method is based on psychological principles.
3. This method suits the learner and the subject.
4. This method create self-confidence in the students.
5. It develops scientific attitude.
6. In this method learner is more active and thus his participation is encouraged automatically.
7. With the help of this method student solves his problem himself.
8. The learnt knowledge by this method is permanent.
9. This method is based on learning by doing.
10. By this method, the teacher can carry whole class with him.

11. In this method, there is very little scope of forgetting.

Demerits of Analytic Method

Analytic Method has following demerits :

1. This method is typical, by which student bore.
2. It is a lengthy and time consuming method.
3. It is not possible to acquire speed and efficiency.
4. It may not be applicable to all topics equally well.
5. This method is not effective at primary level.
6. Every teacher can not use this method successfully.
7. The whole syllabus of mathematics can not be completed within the certain period.
8. This method is not fruitful for below average students.

Conclusion

On the basis of above decision, we can say that analytic method is one of the important method of teaching mathematics. It is accepted that a new problem can be solved on the basis of previous knowledge, thinking, reasoning and insight. So, in analytic method we start to discover the solution of the problem from its whole form.

6. SYNTHETIC METHOD

'Synthetic' word derived from the word 'synthesis' which means to place together things that are apart. This method is opposite to analytic method. In this method we proceed from 'known to unknown'. It starts with something already known and connects that with the unknown part of the statement. It starts with the data available or known and connects the same with the conclusion. Thus, it is the process of putting together known bits of information to reach the point where unknown information becomes obvious and true.

Steps of Synthetic Method

It has three steps :

1. Knowledge on the basis of Problem–Each problem has some base. So in this method teacher firstly gets understands the base of problem to the students.

2. Knowledge of Problem–In this step, teacher imports the knowledge of problem to the students.

3. Solve the Problem–In this step students solve the problem with the help of teacher and explain it broadly.

For example (1)—

If $\frac{a}{b} = \frac{c}{d}$ then proof that

$$\frac{ac - 3ab}{b} = \frac{c^2 - 3ad}{d}$$

$\therefore$ For knowing synthetic

$$\frac{a}{b} = \frac{c}{d} \text{ (we know)}$$

Subtract $\frac{3a}{c}$ from both sides

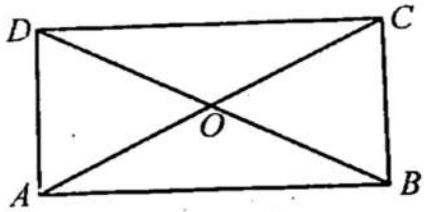
then, $$\frac{a}{b} - \frac{3a}{c} = \frac{c}{d} - \frac{3a}{c}$$

taking LCM and solving

$$\frac{ac - 3ab}{b.c} = \frac{c^2 - 3ad}{c.d}$$

$\therefore$ $$\boxed{\frac{ac - 3ad}{b} = \frac{c^2 - 3ad}{d}}$$

Example (2) : To prove that the diagonals of a parallelogram bisect each other.



Given : A parallelogram $ABCD$ whose diagonals AC and BD interest each other at a point O.

To prove : $AO = OC$ and

$BD = OD$

Proof : In triangles AOB and COD

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\angle OAB = \angle OCD$ (Alternating angles)

and $AB = CD$ (Opposite sides of a parallelogram)

$\Delta AOB \cong \Delta COD$

$\therefore$ $AO = OC$

and $BO = OD$ proof.

Merits of Synthetic Method

The merits of synthetic method are :

1. It is a psychological method of teaching.
2. This method is easy, small and systematic instead of other methods.
3. With the help of this method we easily learn the dull students.
4. It is informative method.
5. It glorifie memory.
6. It is suitables for majority of the learners.
7. This method takes less time and power.
8. By this method student learns more with in short time.
9. It omits the trials and errors like in analysis.
10. It follows the same process as given in the text books.

Demerits of Synthetic Method

Synthetic Method has some demerits :

1. It makes the learner a passive listener.
2. Given knowledge is not permanent.
3. The recall of each step can not be possible for every child.
4. It does not provide full understanding.
5. By this method, it is difficult for a teacher to carry whole class with him.
6. It creates many doubts in the mind of the child.
7. There is no scope of discovery and thinking in this method.
8. The home work assigned after teaching by this method may likely to be difficult for the learner.
9. It does not give full satisfaction to the child.
10. It does not suit the learner and the subject.

Conclusion

Synthetic is the method of formulation, recording and presenting concisely the discovered solution omitting the trial and error. It leads to rote memory and doing by mere limitation. Thus analysis is the process of discovering the solution and synthesis is the method of setting out the solution in a concise form so as to convince yourself and others. Synthetic without analysis is dogmatic. But synthetic is after analysis has a place in the class-room.

Therefore, mathematics teacher should use both analytic and synthetic methods together. In teaching arithmetical problems and geometrical constructions, analysis will only help us to find out a solution. Analysis helps in understanding and synthetic helps in, retaining knowledge.

Difference between Analytic Method and Synthetic Method

Analytic Method	Synthetic Method
1. It is based on the process of analysis.	1. It is based on the process of synthesis.
2. It is logical centred.	2. It is memory centred method.
3. It is informal method.	3. It is a formal method.
4. It is based on inductive reasoning.	4. It is based on deductive reasoning.
5. It is a general method.	5. It is a specific device.
6. It is psychological in nature.	6. It is unpsychological in nature.
7. It proceeds from the unknown to the known facts.	7. It proceeds from the known to unknown facts.
8. In this method teacher and pupil both are active and solution of the problem is found out with the help of pupil.	8. In this method solution is told by the teacher. Here no help is needed from the learners and their work is only to know the cooked solution.
9. It is based upon heuristic approach.	9. It is not based upon heuristic approach.
10. The students can recall and reconstruct easily any steps if forgotten.	10. It is not easy to recall or reconstruct any forgotten steps.
11. It helps in developing the thinking, reasoning and decision making powers of the students.	11. It is helpful only to develop the power of memory.
12. In this method there is a close contact between teacher and taught.	12. In this method there is less contact between teacher and taught.
13. Subject matter becomes solid and durable for a longer time.	13. It is time being and the content is not durable for a longer time.
14. Approach is scientific in nature.	14. Approach is unscientific in nature.
15. Proof can be easily recollected if forgotten.	15. Once forgotten proof can not be recollected.
16. It makes students self-confident and self-reliant.	16. Student does not become self-confident and self-reliant.
17. It is the fore runner of synthesis.	17. It is the follower of analysis.

Relationship between Analytic and Synthetic Method

According to **Arthur Schtze,** "Analysis is the method of discovery, synthesis is the method of concise and elegant presentation."

Analytic method can be fruitful if it is complemented by synthetic method. The analytic method analyses the problem into various sub-parts and these parts are recognised and already learnt facts are used to connect the known with unknown. It puts more stress on reasoning aims of teaching of mathematics. Therefore, this method is suitable for teaching most of the topics of mathematics.

In synthetic method we proceed from known to unknown and combine together a number of facts, perform certain mathematical operations and arrive at a solution. It is concise, time saving and neat method. But it is not suitable for all students. It suits to average teachers and average students. The synthetic proof is shorter but it lacks proper explanation of each step.

Analysis is often identified with induction any synthesis with deduction. Both analysis and synthesis are required in induction as well as in deduction. Thorndike seems to think that all thought, at any rate, all the highest intellectual performance of the mind is analysis. Thus, analytic and synthetic methods are complementary and inter-dependent. One method is incomplete without the another. Therefore, to make the teaching effective both the methods should be applied together. Analysis helps in understanding and synthesis helps in retaining knowledge. Analysis forms the beginning and synthesis forms the follow up work. Therefore, the students should be encouraged through analytic method to cooperate with the teacher and then teacher should use synthetic method to provide knowledge.

7. INDUCTIVE METHOD

Induction means establish a universal truth by showing that if it is true for a particular case and is further true for a reasonably adequate number of cases then it is true for all such cases. In inductive method the rules and formula are established after extensive study of experiences, experiments and examples. This method is more useful in lessons where principles, rules, definitions, generalizations and casual connection between facts are to be established. In inductive method child himself attains the knowledge of some formula or principle with the help of facts, examples and experiments. So the knowledge attained by this method becomes solid and durable and different mental power of the child can also be developed. It is a very effective method.

According to **Joyce,** "Induction is the legitimate derivation of universal laws from individual cases."

According to **Ladul,** "Whenever, we present the facts, examples or things infront of students and we try to get our own expected answers, then it called inductive method."

According to **W.M. Ryburn,** "In the inductive method, the pupil observes, or has his direction drawn to a number of particular instances and then on the basis of those particular instances, he forms a general rule."

Hence, in this method rules and formulae are not supplied by the teachers of the students. Thus, inductive method of teaching leads us from known to unknown, particular to general or example to general rule and from concrete to abstract.

Steps of Inductive Method

In this method four steps are followed :

1. Presentation of specific examples.
2. Observation
3. Generalization
4. Testing and Verification

1. Presentation of specific examples–In this step teacher presents many examples of same types before the students and the solutions of those examples are obtained with the help of students.

2. Observation–In this step students observe the various presented examples minutely and try to see relationship among them and seek some general elements of finding.

3. Generalization–After observing the examples, the teacher and student decide some common formulae, principle or law by logical mutual discussion.

4. Testing and Verification–After deciding some common formulae, principles, student test and verify the laws with the help of other examples. In this way student logically attains the knowledge of inductive method by following above given steps.

Examples 1. Find out S.I. of Rs. 600 at 8% per annum for 3 years.

Solution–

Step first : S.I. of Rs. 100 for 1 year = Rs. 8

S.I. of Rs. 1 for 1 year = Rs. $\frac{8}{100}$

$\therefore$ S.I. of Rs. 600 for 1 year = Rs. $\left(\frac{8}{100} \times 600\right)$

$\therefore$ S.I. of Rs. 600 for 3 years = Rs. $\left(\frac{8}{100} \times 600 \times 3\right)$

= Rs. 144

Second step :

Teacher : What is S.I. for 3 years in above example.

Student : S.I. = Rs. $\left(\frac{8}{100} \times 600 \times 3\right)$

= Rs. 144

Third step :

S.I. for 3 year in above example are—

= Rs. $\left(\frac{8}{100} \times 600 \times 3\right)$

or $\text{S.I.} = \frac{\text{Rate}}{100} \times \text{Principal} \times \text{Time}$

or $\text{S.I.} = \frac{\text{Rate} \times \text{Principal} \times \text{Time}}{100}$

or $\text{S.I.} = \frac{P \times R \times T}{100}$

where, S.I. = simple interest

Fourth step : Students will be able to verify the derived formula by solving other problems of simple interest based on this formula.

Example (2) : Find out the simple interest on Rs. 1200 at the rate of 8% yearly for 18 years.

Solution : Formula $\text{S.I.} = \frac{\text{Rate} \times \text{Principal} \times \text{Time}}{100}$

$= \text{Rs.} \left(\frac{1200 \times 8 \times 18}{100}\right)$

$= \text{Rs. } 12 \times 8 \times 18$

= Rs. 1728

Merits of Inductive Method

Inductive method has many characteristics or merits in mathematics teaching, same are as follows :

1. It is a logical method.
2. This method increase for research.
3. It gives the opportunity of active participation to students in the discovery of formula.
4. This method is very useful for lower classes.
5. It reduces the burden of homework.
6. From this method it is easy to understand a mathematical principle and formula.
7. It is a scientific method, because knowledge attained by this method is based on real facts.
8. This method develops curiosity and interest in the students to learn mathematics.
9. It is based on actual observation, thinking and experimentation.
10. It is a psychological method because many important principles of psychology are used in this method.

Demerits of Inductive Method

Following demerits of inductive method are :

1. This method is useful only small children not big students.
2. This method is more time consuming and it needs much labour.
3. This method is not useful for advanced level of teaching because unnecessary details and explanations may make teaching dull.
4. Only an experienced teacher can use this method successfully.
5. Its application has to be restricted and confined to understanding of rules in the early stage. Once a formula has been established, time should not be wasted in rediscovering it for every subsequent problem.
6. It is neither easy for teacher nor for students to select or present real examples for generalisations.

8. DEDUCTIVE METHOD

This method is opposite to inductive method. Deduction is the process by which a particular fact is derived from some general, known truth. This method is mainly used in Algebra, Geometry and Trigonometry because different relations laws and formulae are used in these sub-branches of mathematics. In the deductive method of teaching learner proceeds 'from general to particular,' 'abstract to concrete' and 'from formula to examples'. In this method help is taken from assumptions, postulates and axioms of mathematics. Therefore, in this method, the learner has to perform only calculation the substituted known values in the given formulae to get the solution of the problem.

According to **Ladul,** "Deductive method teaches definitions and rules at first and then meanings are explained carefully. At the last, facts are used and thus clearly explained in whole."

According to **Joseph Landon,** "Deductive teaching secures first the learning of definion or law or rule, then carefully explains its meaning and lastly illustrates it fully by applying to facts."

According to **Unnati Vishnoi,** "Deductive method is that method in which teacher explains principles first then he represents the relative example to explain those principles."

Steps of Deductive Method

Their are four steps—

1. Knowledge of theories
2. To give theories or examples
3. Derive Conclusion
4. Proof

1. Knowledge of Theories–In this method it is essential that teacher should have knowledge of that theories, which are present in front of the students.

2. To give Theories or Examples–When teacher asks the theories to the students then he gives examples to proof that theories.

3. Derive Conclusion–With the help of examples student derive the conclusion of the theory.

4. Proof–On the basis of conclusion student tests the truth.

Example (1) : Find out the area of rectangle, if length and breadth is 15m and 8m. respectively.

Solution : ∵ Area of rectangle = Length × Breadth

Length = 15 m

Breadth = 8 m

So, $= 15 \times 8$

$= 120\ m^2$

Area of rectangle = 120 square meter.

Example (2) : Find out $(4a + 2b)^2$

Solution : To solve the given problem, students will be told the formula

$(a + b)^2 = a^2 + b^2 + 2ab$ for solving this problem.

or (First variable + second variable)^2 = (First variable)^2 + (second variable)^2 + 2 (First varible × Second variable)^2

Thus,

$(4a + 2b)^2 = (4a)^2 + (2b)^2 + (2 \times 4a \times 2b)$

$= 16a^2 + 4b^2 + 16ab$

Similarly while solving linear equations, teacher tell in advance that for solving it we arrange unknown quantities on one side and known quantities on other side and while transposing the quantities the sign of quantities change.

Merits of Deductive Method

The followings are the main advantages of this method :

1. This method increases memory power.
2. By using this method the speed of gaining knowledge increase because students directly use the formulae for solving the problem.
3. It is short and time saving method.
4. At the practice and revision stage, this method is adequate and advantageous.
5. It combines with the inductive method to remove the incompleteness and inadequacy of the later.
6. Laws, principles and formulas can easily be checked by using this method.
7. This method is used for teaching theorem and axioms of geometry tables in Arithmetics etc.

Demerits of Deductive Method

Some demerits of deductive method are as follows :

1. This method is not useful to develop scientific attitude in the students.
2. It is very difficult for a beginner to understand an abstract formula if it is not proceded by a number of concrete instances.
3. This method is only useful if it is combined with inductive method.
4. It is not suitable for the development of reasoning, logical thinking and scientific attitude.
5. The students can not become active learners.
6. Memory become more important than understanding and intelligence, and that is educationally unsound.
7. By using this method the teaching-learning process becomes uninteresting and dull.
8. In this method student can not understand without examples.

Difference between Inductive and Deductive Method

Inductive Method	Deductive Method
1. This method is based on 'from known to unknown', 'from particular to general', from concrete to abstract' and 'from example to formula' maxims of teaching.	1. This method is based on 'from formula to example', 'from general to particular', 'from abstract to concrete' maxims of teaching.
2. In this method students are active.	2. In this method teachers are active.
3. This method is helpful for generalisation.	3. This method is not capable to generalisation of students.
4. This is a path of research.	4. It is a path of imitation.
5. It is a psychological method.	5. It is a unpsychological method.
6. It is a slow method and all knowledge is to be acquired first hand by the pupil.	6. It is a quick method. The pupil awaits opportunity of getting finished knowledge acquired by others.
7. It is useful for the beginners.	7. It is useful for students of higher classes.
8. By this method, a habit of discovery is developed in students.	8. By this method, a habit of discovery is not developed in students.
9. In this method every step is important and students learn to write them.	9. In this method students do not learn to write so many steps.
10. This is understanding centred.	10. This is memory centred method.
11. This method gives emphasis on original and creative work.	11. This method gives emphasis on problem-solving.
12. Teaching-learning process become interesting and effective by the use of this method.	12. Teaching-learning process become dull by the use of this method.
13. Mathematically it is – much training + little information.	13. Mathematically it is – much information + little training.

Relationship between Inductive and Deductive Method

According to a **Scholar**, "In all natural development practice precedes theory and a working notion is the germ of exact conception ..... An exact

concept is the final goal and is the outcome of careful analysis of the working notion, thus making the thought more accurate and practice more rational."

Deduction is a handmaid of induction, and what is left undone in induction is completed by deduction. These two methods are friendly partners so that the short comings of the one are offset by the other. Indeed inductive and deductive methods are supplementary to each other. The main basis of inductive method is origin and development of knowledge, while perceptual presentation is the main basis in deductive method. So inductive method is a forerunner and deductive method is its follower. Hence the process of establishing the formulae and laws and then proceeding ahead and practising is known as Inductive-Deductive Method.

Thus induction and deduction are not two opposite things; these are only two different modes of thinking and one is supplement of other. We cannot use one method to the exclusion of the other as these are inseparable. Therefore, teaching should begin with induction and end in deduction. So, In practice, a mathematics teacher uses both methods as a combination.

9. PROJECT METHOD

This method is widely used in mathematics teaching. Project method is based on philosophy of pragmatism given by John Dewey. His disciple **Sir William Kilpatrick** propounded this method. This method consists chiefly of building a comprehensive unit around an activity which may be carried on in the school or outside. This method aims to make learning effective and to give student the real training for life. It emphasises 'Learning by doing' and 'learning by living' both.

It is based on the belief that the students learn more through relation, comperation and activity. In this method the data is collected keeping the aim in mind. The students do work according to their will in natural circumstances. So that they can learn to live their life in decant way without any problem. This method always has an aim and the project is done. To complete the project process is framed as the result of which the students complete the project in natural circumstances according to their will and interest.

Many scientists have defined in the following ways :

According to **Kilpatrick,** "A project is a whole hearted purposeful activity proceeding in a social environment."

According to **Stevenson and Charters,** "A project is a problematic act carried to completion in its natural setting."

According to **Thomas and Lang,** "Project is a voluntary undertaking which involves constructive effort and eventuates into objective results."

According to **Parker,** "It is a unit of activity in which pupils are made responsible for planning and purposing."

According to **Ballard,** "A project is a bit of real life that has been imported into school."

On the basis of above definitions we can say that :

1. Project is related to some activity in which the students are always active.
2. Project is the selected work by the student not others.
3. Without aims project can not be completed.
4. Its base is psychological which is according to the student's age, interest, attitude and aptitude.
5. This method is inspiring. After completing one step the student moves to other one. The success at the first step inspires the student to move on the second.

Concept of Project Method

It has the following concept :

1. Learning process goes towards a certain aim to achieve it.
2. The student designs the process himself.
3. The evaluation of the achieved aim is also possible to ascertain that how far the aim has been achieved.
4. The student understands himself in this method.

Types of Project Method

It is of the two types :

1. Individual Project–The students completes the project himself. But it does not develop social qualities.

2. Social Project–In this method students of the class complete the project collectively. It develop social qualities in the students.

Steps of Project Method

Following steps are involved in project method :

1. Creation of situation
2. Choosing the project
3. Planning the project
4. Executing the project
5. Evaluation
6. Recording

1. Creation of Situation–At first stage of project method, the teacher should know the interest, attitude and aptitude of the students. Then, he must create situations according to their interest by means of conversation on various topics, discussions of picture, buildings, cities etc., stories lives of greatmen, excursion etc. Then situation is provided to students to study the different problems from different angles and see the desirability of solving any specific problems for themselves.

2. Choosing the Project–The teacher helps the student in choosing a situation. Teacher presents various projects to the students. The proposed project should be according to the feal need of students. The purpose of project should be common to whole class and acceptable to all.

3. Planning the Project–After the selection of the project the student himself makes the plan. The success of the project depends on it. A good plan must have following considerations:

– Material and equipments essential for the project.
– Time limit and estimated cost.
– Resources available.
– Division of work and duties to be assigned.

The teacher guides the students time to time but does not impose his ideas on them.

4. Executing the Project–This step involves various kinds of activities such as collecting informations, discussions, reading and writing, visiting different places and people, consulting reference books, drawing maps etc. All work is done in the guidance of teacher.

5. Evaluation–To take decision on the completed work is called the evaluation. The teacher and the students evaluate the project. It is very important because if there was any mistake in the project then it can be found out. Thus the whole project is reviewed by the student under the guidance of teacher.

6. Recording–It is an important step of the project method. The report of all the works from the selection to the execution of project is prepared and recorded. Here students also record their experiences. These records are useful for future references.

Role of Teacher in a Project

The teacher plays a very important role in the project method which are :

1. It is very sympathetic towards the students.
2. The teacher should provide democratic atmosphere in the classroom.
3. Time to time he encourages the students.
4. The teacher should work as a friend and guide.
5. The teacher should have thorough knowledge and experiences.

Some Project in Mathematics

- Study of School budget.
- Execution of hostel mess.
- Collection and study of the currency of different countries.
- Uses of mathematics in large and small business.
- Bus and train fares.
- Collection of data regarding population, death rate, birth rate etc.

Merits of Project Method

The followings are the major merits of this method :

1. It is a psychological and scientific method.
2. It develops the students physically and mentally.
3. In this method students are free to choose any project.
4. It provides opportunity for mutual exchange of ideas.
5. It develops self-confidence and self-discipline in the students.
6. It promotes social habits and discipline useful for real citizenship.
7. It upholds the dignity of labour.
8. It introduces democracy in education, because it necessitates co-operation among students and their acting together for a common cause.
9. It emphasises problem solving rather than cramming.
10. It develops group interaction.
11. In this method every student gets equal opportunity for development.
12. It provides motivation.
13. It brings about concentration of studies and correlation of activity and subjects.

Demerits of the Project Method

Besides so many merits or advantages it also involves certain demerits. The following are the main drawbacks of this method:

1. It is an expensive method.
2. Whole syllabus can not be tought through by this method.
3. It requires a lot of material not easily available so, it is not suitable method for ordinary schools.
4. There is no saving of time, energy and effort.
5. It is likely to prove costly.
6. The knowledge is not acquired in a sequentials and systematic manner.
7. It can not follow the time-table.
8. In this method the teaching and learning become disorganised, irregular and discontinuous.

9. This method is not suitable for a class having a large number of students because a teacher can not guide or supervise so many projects properly.
10. This method needs well decorated and equipped laboratory which is not available in every schools.
11. There are certain practical difficulties in the application of this method in Indian situation such as lack of resourses, wide curriculum, limited time, large number of students in a class, non-availability of text-books and reference books etc.

Conclusion

The approach of this method is scientific and psychological. This method brings life to the school atmosphere. As it is not suitable for drill and continuous and systematic teaching, it is not very desirable to use it freely. It leads to understanding and develops the ability to acquire and apply the knowledge in actual situation. But it is not very much suitable for our schools because there are so many difficulties in its use. This method cannot be used as a regular method of teaching of mathematics because there is no sufficient drill work which is essential for the mastery of a topic in mathematics.

Thus, teacher should select certain projects which can be undertaken in schools without upsetting regular teaching. He should also work as a careful guide during the execution of the project.

10. PROBLEM-SOLVING METHOD

This method is more important than any other method of mathematic teaching, because in it the student, himself select a problem and solve it. Problem solving method is mainly concerned with the training of students to overcome the difficulties and obstacles occurred in the way of solving the problems.

Human life is full of problems and everyone has to face them. To reach a specific goal of life, one has to attack the overcome these problems. A child has to meet and solve various problems which present themselves in his physical, intellectual emotional and social life. These problems grow in number and complexity as he grows older and older. His success in life depends upon his ability and competency to solve them. Therefore, it is essential that children are to be trained in problem solving, so that they can lead their life smoothly.

The problem-solving method aims at presenting the knowledge to be learnt in the form of a problem. It begins with a problematic situation and consists of continuous, meaningful, well-integrated activity. Mathematics

is a subject of problems. Its teaching and learning demands solving of innumerable problems. Efficiency and ability in solving problems is a guarantee for success in learning this subject.

This method has been developed from pragmatism. It is just like Heuristic method. It develops logical and observation power in the students.

Definitions

In the definitions of problem-solving method used are interprete many words as creative-thinking, concept formation, insight, logical thinking etc.

According to **Gates and others,** "Problem-solving is a form of learning in which the appropriate response must be discovered."

According to **Skinner,** "Problem-solving is the frame work or pattern in which creative thinking and reasoning takes place."

According to **C.V. Good,** "In Problem-solving method a student is inspired to learn by creating challenges. It is a specical method, in which a big problem is solved by small but group solution of relating problem."

According to **Yokam and Simpson,** "A problem occurs in a situation in which a felt-difficulty to act is realised. It is a difficulty that is clearly present and recognised by the thinker. It may be a purely mental difficulty or it may be physical and involve the manipulation of data. The distinguishing thing about a problem however is that it impresses the individual who meets it as needing a solution. He recognises it as a challenge....."

According to **L.A. Averill,** "The only worthwhile life is a life which contains its problems, to live without any longings and ambitions is to live only half-way."

According to **Ausubel,** "Problem-solving involves concept formation and discovery learning."

In shortly we can say that, it is a process of raising a problem in the minds of students in such a way as to stimulate purposeful reflective thinking arriving at a rational solution. There elements seem to be involved here : a situation which presents some difficulty, perplexity or doubt requiring solution, a goal or an end involving some aspect of the situation for which no ready answer can be given and desire or motive that stimulates an attempt to find the answer.

This method is used according to the problem of the students. When a student does not have a solution for the present problem and he has to solve it then collection aid to solve this problem. He finds many problems in doing it but finally he is successful in solving the problem. This method is called problem-solving method.

Thus, in this method mental activity or ability and solution are very important. Every effort of the student make to solve the problem is important.

Steps of Problem-Solving Method

Following Steps are involves in the Problem-Solving Method :

1. Selection of Problem.
2. Presentation of the Problem
3. Collection and Organisation of facts.
4. Classifying the importance of the problem
5. Evaluation of facts and decisions about possible solution.
6. Generalisation and conclusion
7. Evaluation of results and preparing records of the problem.

1. Selection of Problem–The selection of the problem should be done by the teacher and student both. The nature of problem should be made very clear to the students.

2. Presentation of the Problem–After selecting the problem, teacher should present the problem well before the student. The teacher should also make it clear that how this problem can be solved and how the related data and informations can be collected to get the solution of the problem.

3. Collection and Organisation of facts regarding Problems–The student should be stimulated to collect data and information in a systematic and scientific manner. The teacher can suggest many points regarding collection of data to the students.

4. Classifying the importance of the problem–When the students know the importance of the problem then they become more interested about it. That is way in this method, they are told the importance of the problem.

5. Evaluation of facts and decision about possible solution–After the collection of the facts it is determined that which feel is related to the problem and can be rejected. After the evaluation of the facts, the problem is tried to be solved. The teacher and the students all try to solve the problem.

6. Generalisation and conclusion–After evaluation the facts are generalised which are helpful to solving the problem.

7. Evaluation of result and preparing records of the problems–Finally the record of the problem is prepared and the solution is evaluated.

Merits of Problem Solving Method

This method has following merits :

1. The students and the teacher together take interest in solving the problem. It enhances work efficiency.

Conclusion

The problem-solving is a suitable approach in the teaching of mathematics. It trains the students in problem solving which may be helpful to them in leading their life smoothly because facing and solving problems is the true nature of human life. It develops in the learners the ability to recognise, analyse, solve and reflect upon the problematic difficulties. Problem-solving method helps us at every step in our teaching-learning process. In this method the teacher should carefully select the problems which are real and have definite educational values and an intellectual atmosphere should be created in the class for problem solving.

6.5 Lecture- Demonstration Method

Lecture-Demonstration Method is considered to be a method superior to lecture method as it combines the advantages of both the lecture method and the demonstration method. In this method both the teacher and the taught are active participants in the process of teaching.

The lecture-demonstration can be effectively used for teaching mathematics. By using this method it is possible to easily impart concrete experiences to students during the course of a lesson, when the teacher wants to explain some abstract points. This method combines the instructional strategy 'information imparting and showing how'. For example, while teaching geometrical constructions the teacher has to explain how to construct the geometrical figure with the given specifications, simultaneously demonstrating how to carry out the construction following the sequence of steps.

In this method, the teacher performs the experiments before the class and simultaneously explains what he is doing. For example, while explaining how to form a right circular hollow cone using a sector cut out from a circle, the teacher has to show how to fold the sector to get a cone so that the radius of the sector becomes the slant height of the cone. He also encourages students' participation by asking relevant questions, requiring them to keenly observe and draw inferences. Afterwards through questioning and cross-questioning, the inferences drawn by the students are discussed in the class.

Merits of Lecture -Demonstration Method

- It is a psychological method as students take active interest in the learning process.
- It is useful for all students of varying abilities.
- It is an economical method as compared to purely student-centred methods.
- It leads the students from concrete experiences to abstract concepts
- It encourages students' participation in learning.
- It trains mental faculties such as power of observation, reasoning and drawing inferences.

Demerits of Lecture -Demonstration Method

- It does not provide first-hand experiences to the students.
- It does not provide for individual differences. It caters to the needs of average students.
- It does not develop manual and manipulative skills and cannot be a substitute for laboratory method.
- If not very attentive, the students fail to observe minute details of the demonstration.
- It is not applicable for higher level mathematics.

Applicability of Lecture – Demonstration Method

Lecture – Demonstration Method is best suitable for teaching mathematics at lower classes. All topics in mathematics do not yield for the use of this method. Construction of geometrical figures and some basic concepts in geometry can be most effectively taught through lecture – demonstration method.

TECHNIQUES AND STRATEGIES OF MATHEMATICS TEACHING

There are numbers of techniques which can be effectively used for the teaching of mathematics. These teaching techniques are such aids which are used to make the lesson interesting, to explain the content and to remember it by heart during teaching-learning process. These techniques are hot the same as a method of teaching. During the course of teaching the teacher thinks of not only the method he is to adopt but he may feel it necessary to adopt some other devices and techniques aiming at effective teaching and learning. These devices are explanation, narration, exposition, illustrations and questioning are commonly used in the teaching of various subjects. Some other devices such as drill work, review, repetitive practices, home work, assignment etc. are make up the mental level of the child.

In mathematics, the techniques of oral, written work, drill work, home work, review, self-study and supervised study etc. can be used to make the learning meaningful. Some important techniques are as given below :

1. ORAL WORK

It is a form of mental work. It plays a very important place in mathematics. In mathematics oral work is not only interesting but may be effective especially in the initial stages. The basic theories as-addition, substration, multiplication, division etc. are done by orally in mathematics. The results of oral work in the saving of time and efforts through omission of certain steps. It gives a quick and easy start to the process of learning. Oral questions can be graded according to difficulty in a better manner. This demands power of careful listening, visualisation, quick thinking and decision making.

The meaning of oral work is that work, in which child calculate any work without help of any pen or pencil and solve the problem.

Oral work is very important for the success of human life. Every person use oral work in a different form. It create the calculating capability quickly. The major part of mathematics has to be covered in written form, but its application lies in oral form. While doing the written work in

mathematics, the oral work precedes it. Thus the oral or mental work is the backbone of not only written work, but also overall works in the mathematics.

Purpose of Oral Work

Teacher use the oral work for the following purposes :

1. To develop the creatively in the students.
2. To develop the concentration in the students.
3. To develop the memory power in the students.
4. Quick evaluation of the lesson.
5. To develop the felling of self-confidence in the students.
6. To develop the habit of thinking in the students.
7. To make up the eassy presentation of new topics.

Advantages of Oral Work

The main advantages of oral work is :

1. It is helpful in the repeatition of lesson.
2. It develops the thinking, understanding and imagination power in the students.
3. Its develop mental alertness and quick thinking.
4. It develops the concentration power in the students.
5. By the help of this technique memory of the child can be tested.
6. It develop the feeling of competition in the students.
7. It helps in diagnosing the difficulties and doubts of students.
8. This technique is very suitable for securing attention of students.
9. It is helpful to maintaining discipline in the class.
10. Revision of the subject matter taught can be done quickly.
11. It can be developed auditory sense in the child.
12. This technique are helpful in developing a lesson and testing their understanding.
13. It has an appeal for the eyes and ears and so it is liked by the students.
14. It removes the shyness and hesitation of children.
15. It helps in elucidation and illustration.
16. It develops the habit of working with speed and accuracy, exactness and precision.
17. This technique can be used for teaching many topics.

Disadvantages of Oral Work

1. It is a very quick device and is not suitable for all the students.
2. All the problems of subject can not be solved orally.

3. Record of the learnt material can not be kept for reference if the work is only carried out orally.
4. Expression and writing power of the students can not be checked properly.

Precautions for using Oral Work in Mathematics

Following precautions should be used for using oral work in mathematics :

1. Should be use short answer type questions in oral work.
2. The language of questions should be simple and clear.
3. The given work should be complete in limited time.
4. After asking the questions teacher should give the time of thinking for the students.
5. The questions should be according to the ability of the students.
6. These questions are arrange in a sequence.
7. It should give more emphasis on exercise and calculation.

From above mentioned values of oral work, it is evident that it occupies an important place in teaching of mathematics. Its appeal for the eye and ear has greater value than insistence on written work in the beginning of a topic or lesson. The learning of various new concepts requires sufficient oral drill. Therefore, the oral mastery of every new idea by the students should be the first concern of every mathematics teacher.

3. DRILL WORK IN MATHEMATICS

Drill work is also important technique in mathematics. Drill work is based on the psychological principles such as learning by doing and law of exercise. Drill is not mere a repetition of an act. It is a serious work activity leading to perfection opportunity of self-improvement. It plays a prominent role in learning because it affords a convenient and fairly efficient medium for the rapid memorization of details and the automatization of processes.

Drill work is an admitted fact that practice makes a man perfect. It is a means to strengthen the knowledge already acquired. This technique is used by the teachers to revise a lesson that has already been taught. It is not the testing of results but the strengthening of learning. It leads to habit formations.

According to Sexsana and Oberai, "The meaning of drill work is to apply the learnt tasks or skills or read material in novel situations. Drill work tells the teacher whether the teaching objective have been achieved or not. In other words, the knowledge gained by drill work gets consolidated."

Purpose of Drill Work

The main purpose of Drill Work are :

1. To develop the self-confidence in the students.
2. To give permanent and clear knowledge about the mathematics to the students.
3. To increase the mental power of the students.
4. To develop the work efficiency in the students.
5. To develop the different habits in the students.

Principles of effective Drill Work

1. The material to be memorised or drilled upon should be meaningful.
2. The achievement of the learner in drill should be frequently tested.
3. Drill should be varied, it should be given in various forms, otherwise it will become monotonous.
4. Drill assignments should not be made tasks, the students should be enabled to take pleasure in them.
5. Moderate initial practice and systematic review are superior to over-learning.
6. Drill should be properly and sufficiently motivated and once initiated, it should be well-organised.
7. Drill exercises should be conducted in such a manner that pupils can work at different rates and at different levels according to their abilities.
8. Drill should follow developmental and discovery stages of learning and be used to reinforce and extend basic learning.
9. Drill should be concentrated upon particular skills or even on particular details of operation.
10. The summary can also be developed by asking questions related to the concept developed in the class-room.

Functions of Drill Work

The function of drill work can be summarised as follows :

1. To fix the material learnt in the mental make up of the child.
2. To develop confidence and a sense of achievement in the student.
3. To develop speed and accuracy in the learning of mathematics.
4. To revise the knowledge already learnt.
5. To develop communicational skill for the learning of mathematics.
6. To involve the students in the learning of details of the topic minutely.
7. To motivate the average and below average students to learn by their errors.

Advantages of Drill Work

Following advantages of drill work are :

1. It is important for teachers and students both.
2. It is beneficial for exercise of lesson.
3. It can prepare clear and permanent knowledge in mathematics.
4. It is very economical device of learning and teaching.
5. It is good technique of learning for beginners.
6. It is possible to prediction of memory of children.
7. Accuracy of learnt material can be improved.
8. Immediate reinforcement through practice and application is desirable.
9. It is a less time consuming technique of learning.
10. Drill leads to habit formation.
11. Mental, physical and emotional outcomes of school activities are always the results of habit formation and hence dependent on drill.
12. The purpose of drill is to fix or to maintain an association connected with the learning process.
13. Helpful to achieving the objectives of mathematics teaching.

Disadvantages of Drill Work

1. Drill work is not suitable for all topics.
2. Drill work creates disturbance in the other classes.
3. It is not effective without good and clear voice.
4. Sometimes drill becomes an exercise in academic futility and no one benefits.
5. Careful questioning by the teacher is usually needed in drill work.

Suggestions and Precautions for doing Drill Work in mathematics

Following points should kept in mind to make drill work more effective :

1. Drill work should be fixed at a limit time.
2. Drill work should be interesting affair and not become unpleasant and boring.
3. Teacher should have patience to give drill work.
4. It should be meaningful and should always be based on the facts taught.
5. Drill work should be sufficiently motivated and well organised.
6. The achievement of students in drill should be frequently tested.

7. Moderate initial practice and systematic review are superior to over learning.
8. The drill work can be made effective and interesting by introducing the elements of play, competition and group work in it.
9. Drill work should be an interesting affair and not become unpleasant and boring.
10. Students should be given sufficient opportunity and proper atmosphere for drill work.
11. Mental level should be according to the children in a drill work.
12. It should be based on previous knowledge.
13. It should be create the positive point of view in the students.
14. Drill exercises should be short and distributed over a period of time.
15. Drill assignments should not be made tasks, the students should be enabled to take pleasure in them.

4. HOME WORK IN MATHEMATICS

Home work has to be given regularly to the students to supplement classroom teaching for more practice and application of rules, theorems and formulas. It is one of the practical techniques of teaching. It is an introduction to study and enabling the learner to become student in the real sense. It becomes all the more important and necessary in view of the heavy load of work as compared to the time available. By giving home work means creating in the children a study environment at home. The nature and amount of home work should be given according to the capacities of the children. Home work should be assessed as a part of internal assessment and proper weightage should be given.

According to **Adams,** "It is a step towards self-education, enabling the child to use his own resources and work unsupervised."

According to some educationists, if school time is used sincerely and efficiently, home work may not be necessary. In general conditions, the child does a good amount of work during school hours. On the other hand, several educationists are of the opinion that 4-5 hours work of school is insufficient and it must be supplemented by some work to be done at home.

Objectives of Home Work

The objectives of giving home work may be summarised as follows :

1. It provides the opportunity of independent work.
2. It supplements the class-room teaching.
3. It acts as a link between parents and teachers.
4. It cultivates the habit of regularity and hard work among the children.

5. It creates an environment of school feeling at home among the children.

Importance of Home Work in Mathematics

Its importance is seen in the following points :

1. It supplement the classroom teaching.
2. Home work is essential because the school hours are not enough for the necessary work.
3. It develops a habit of regularity and hard working.
4. It brings about closer relationships among the parents and the school.
5. It may prove as a link for teacher-parent cooperation.
6. It is useful to utilize the leisure time of the students.
7. It develops the habit of independent working in the student.
8. It develops self-confidence and self-reliance amongst the children.

Disadvantages of Home Work in Mathematics

Some disadvantages of home work in mathematics are as follows :

1. It may adversely affect the health of the children.
2. It do not provide suitable conditions for work.
3. It also deprives the children of their leisure time.
4. Many people involve their parents or others to complete their home work.
5. It takes too much time of the children after the school.

Precautions and Suggestion of Home Work in Mathematics

The home work may serve a useful purpose, if following precautions and suggestions are followed while assigning the home work :

1. Home work should not be considered as a sort of punishment.
2. Home work should be given in brief, so that the pupils will be more willing to try to complete it.
3. It should neither be too easy nor too difficult.
4. Do not assign the problem to the children discriminately.
5. It should not be given by the teacher in a disturbed state of mind.
6. Home work should not be use as a punitive device.
7. It gives clear directions to the students.
8. Home work should be duly checked and corrected. If it is not checked, the students may fall into the bad habit of evading it.
9. Home work time-table should be prepared in advance. Each subject must get its due share of home work.
10. While assigning home work, the home conditions of individual students must be kept in mind.

7. ASSIGNMENT IN MATHEMATICS

The main aim of assignment to is provide some work to do. It is the work assigned to the student, which may be done by the students at home or at school. An assignment may be a revision of the lesson learnt or a sort of preparation for the lessons to be learnt. It may consists of solving a few mathematical problems, understanding a proposition or a group of propositions, solving riders based on a proposition, making many constructions of the same type, preparing illustration for a topic, collecting mathematical data from the environment, finding out application of mathematical knowledge, tracing out historical background of a mathematical concept, formulating problem on a topic, carrying out a mathematical project, elucidation of an intricate mathematical concept etc. According to this assignment must be based on the activities and interests of the students. It is on complex problems, learning of new skills or where students are just new to that type of learning, should be carried on under the supervision of the teacher, otherwise they may frustrate the students.

Characteristics of Assignment

The main characteristics of assignment method are :

1. A good assignment should have clarity and definiteness.
2. It should remove the problems and difficulties of the child.
3. It should be correlated with previous knowledge and experiences.
4. It should give proper reinforcements to the child.
5. It should be motivating and interesting.
6. It should be directing and stimulating the learning activity.
7. It should be based on individual differences.

Preparation of Assignment

For preparing effective assignment following points should be kept in mind :

- Reference to previous learning.
- Discussion leading to new learning and activity.
- Acceptance of activity and anticipating difficulties.
- Removal of difficulties.
- Planning for the material to be used.
- Details of the task to be done.
- Carrying out the assignment.
- Overseeing and checking the work.

Purposes of Assignment in Mathematics

- To Prepare illustration for a topic.
- To carry out some mathematical projects.
- To solve mathematical problems
- To create interest in mathematics.
- To correlate the experiences and previous knowledge of the child.
- To develop the habit of practice.

- To formulate problem on a concept or topic.
- To develop the skills of problem-solving.
- To apply the mathematical knowledge in solving the problems.

Suggestions for preparing effective assignment in mathematics

1. The assignment should preferably arise out of the activities, needs and interests of the pupils.
2. An assignment should be insightful.
3. The purpose of assignment should be definite.
4. The assignment should be a cooperative activity in which the teacher and the pupils take an effective part.
5. Individual differences must be considered at the time of assignment.
6. Weekly assignments are preferable to daily or monthly ones.
7. The task of assignment should be pinpointed be cause vague and lengthy assignments are of no use of achieve better results.
8. Interaction between pupil and teacher is must in an assignment.
9. It must be motivative, clear up doubts and misunderstandings.
10. Reference books, text-books and other teaching-learning material should always be used to stimulate and direct the pupils assignment activities.
11. The assignment should be well prepared with a proper planning. The subject and material should be according to the need and level of students.

9. REVIEW IN MATHEMATICS

It is a one of the important and effective fixing devices in mathematics. It is the mental process of 'going over' the material already learnt. The aim of review not only at the fixation and retention of details but also at the thoughtful organization of the important things in a unit or a chapter in order that the relationship of the various parts to each other and to the while unit may be understood clearly.

The review recalls and renews and establishes new relationships. It is the process of reworking of the learnt material. It is broader and more effective or inclusive as compared to the drill. It involves not only reviewing, but also rethinking, regrouping, reconstruction, rearrangement and reunderstanding. It takes place under the active supervision and guidance of the teacher.

Purpose of Review

The main purpose of review are :

1. To fix the knowledge.
2. To develop new interest in old material.
3. To prepare background for new learning.
4. To have better understanding.
5. To introduce new elements and recoganisation of thoughts.

Suggestion for the Success of Review

The following suggestions may be helpful for effective review :

1. Review is not a repetition but it also involve new learning.
2. It should be directed at the weak points or doubts of the students.
3. Graphical, symbolic, pictorial etc. representations are very suitable for the purpose of review.
4. It should be focus on the main points.
5. Exercises given at the end of the chapters of the text-books may be used for reviewing.
6. Review may take the form of verification, checking of the learnt material.