

PEDAGOGY OF MATHEMATICS

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UNIT TEST 1

PART - A

FIRST YEAR 2019

1. Give atleast three definitions of mathematics and describe the nature of mathematics with examples.

Definitions:-

"Mathematics is a science dealing with the logic of quantity and shape and arrangement."

- Word Net Dictionary

"Math is defined as conjecturing and proving theorems in first-order logic."

- Joe Shipman

"Mathematics is commonly defined as the study of patterns of structure, change and space."

- wikipedia encyclopedia

Nature of Mathematics:-

The Nature of Mathematics is made explicit by discussing it under the followings.

* Mathematics : A Science of Discovery

* Mathematics : An Intellectual Game

* Mathematics : The Art of drawing conclusions

* Mathematics : A tool Subject

* Mathematics : A system of logical processes

* Mathematics : An Intuitive method.

Mathematics : A Science of Discovery

* Mathematics is the discovery of relationships and expression of those relationships in symbolic form in words, in numbers, in letters, by diagrams or by graphs (E.E. Biggs, 1963)

(2)

* According to A.N. Whitehead (1912) "Every child should experience the joy of discovery." Initially a child's discoveries may be observational. But later, when its power of abstraction is adequately developed.

* Today, it is discovery techniques, which are making spectacular progress. They are being applied in two fields; in pure number relationships and in every problems, involving such things as money, weights and measures.

Mathematics: An Intellectual Game

* It can be treated as intellectual game with its own rules and without any relation to extend criteria.

* Mathematics is mainly matter of puzzles, paradoxes, and problem solving - a sort of healthy mental exercise.

Mathematics: The Art of Drawing conclusion

* One of the most important functions of the school is to familiarise children with a mode of thought which helps them in drawing right conclusions and inferences.

* According to J.W.A.,

i) Learner should know whether or not he has drawn the correct conclusion.

ii) The learner to begin with simple and very easy conclusions to pass in well graded sequence to very difficult ones, as the earlier ones

iii) The introductory subject be found in the other subjects also, and in human interactions, in general.

Mathematics: As a tool subject

* It could be more elegantly expressed as "Mathematics - handmaiden to the sciences". From the beginning, down to the nineteenth century, mathematics has been assigned the status of a servant. Then in the nineteenth century, mathematics attained independence.

* Mathematics has its integrity, its beauty, its structure and many other features that relate to mathematics as an end in itself.

* However, many conceive mathematics as a very useful means to other ends, a powerful and incisive tool of wide applicability.

Mathematics: A system of logical process

* Polya suggested that mathematics actually has two faces.

i) One face is a 'Systematic deductive science'. By defn, undefined terms, axioms and theorems.

ii) Mario pieri stated "Mathematics is a hypothetico - deductive system."

iii) Benjamin pierce - 'The science which draws necessary conclusions'.

Mathematics : An Intuitive Method

* Intuition implies the act of grasping the meaning or significance or structure of a problem without explicit reliance on the analytic apparatus of one's craft.

* It is the intuitive mode that yields hypothesis quickly.

* It precedes proof : it is what the techniques of analysis and proof are designed to test and check.

* It is a form of mathematical activity which depends on the confidence in the applicability of the process rather than upon the importance of right answers all the time.

* Intuition when applied to mathematics involves the concretisation of an idea not yet stated in the form of some sort of operations or example.

1. These are the various nature of mathematics which deals with the Mathematics subject. And also, some of the nature gave very brief about the mathematics.

2. Explain briefly the contribution of Indian Mathematics towards the development of mathematics.

Contribution of Mathematicians:-

The height which mathematics is occupying today and the progress, which it has made through the ages, is all due to the dedicated and sustained work of its great students who become immortal because of their contribution to mathematics. Some of the Indian Mathematicians are given below,

- * Aryabhata
- * Bhaskaracharya
- * Srinivas Ramanujan
- * Brahmagupta
- * Mahavira
- * Varahamihira

Aryabhata:-

* First we look at the system for representing numbers, which Aryabhata invented and used in the Aryabhatiya. It consists of giving numerical values to the 33 consonants of the Indian alphabet to represent 1, 2, 3, ..., 25, 30, 40, 50, 60, 70, 80, 90, 100. Vowels are denoted by 100, 10000, ...

In fact the system allows numbers up to 10^{18} to be represented with an alphabetical notation.

* Aryabhata gave an accurate approximation for π . He wrote in the Aryabhatiya the following:

$\pi = 62832 / 20000 \approx 3.1416$ as surprisingly accurate value.

In fact, $\pi = 3.14159265$ correct to 8 places

* We now look at the trigonometry contained in Aryabhata's treatise. He gave a table of sines calculating the approximate values. In order to do this he used a formula for $\sin(n+1)x - \sin nx$ in terms of $\sin nx$ and $\sin(n-1)x$. He also introduced the Versine ($\text{versin} = 1 - \cosine$) into trigonometry.

Bhaskaracharya:-

* Bhaskaracharya became head of the astronomical observatory at Ujjain, the leading mathematical centre in India at that time. In many ways Bhaskaracharya represents the peak of mathematical knowledge in the 12th century.

* Bhaskaracharya studied Pell's equation $px^2 + 1 = y^2$ for $p = 8, 11, 32, 61$ and 67 . When $p = 61$, he found the solutions, $x = 226153980$, $y = 1776319049$. When $p = 67$ he found the solutions $x = 5967$, $y = 48842$.

* The topics covered in the thirteen chapters of the book are: definitions; arithmetical terms; interest; arithmetical and geometrical progressions; plane geometry; solid geometry; the shadow of the gnomon; the kuttaka; combinations.

* Bhaskaracharya, like many of the Indian mathematicians, considered squaring of numbers as special cases of multiplication, which deserved special methods.

Mahavira

* The only known book by Mahavira is Granita Sara Samgraha, dated 850 AD, which was designed as an updating of Brahmagupta's book.

* The Granita Sara Samgraha by Mahavira is the earliest Indian text, which we possess and is devoted entirely to mathematics.

* Regarding the development of a place-value number system is Mahavira's description of the number 12345654321 which he obtains after a calculation.

* Among topics Mahavira discussed in his treatise was operations with fractions including methods to decompose integers and fractions into unit fractions.

* Mahavira gave special rules for the use of permutations and combinations, which was a topic of special interest in Jaina mathematics.

* Mahavira also attempts to solve certain mathematical problems, which had not been studied by other Indian mathematicians.

* The formulas for a conch-like figure have so far been found only in the works of Mahavira and Narayana.

Srinivasa Ramanujam:-

* He studied the special form, structure and distribution of composite numbers.

* He made the statement that every even integer greater than two is the sum of two primes.

* Investigation on divergent series.

* He was one of the greatest masters in the field of definite integrals and elliptic functions.

Brahmagupta:-

* Brahmagupta's understanding of the number systems was far beyond others of the period. He developed some algebraic notation. He gave remarkable formulas for the area of a cyclic quadrilateral and for the lengths of the diagonals in terms of sides.

* Brahmagupta also studied arithmetic progressions, quadratic equations, theorems on right-angled triangles, surfaces and volumes.

* One has to wonder whether Brahmagupta's second value for the length of the year is taken from Aryabhata since the two agree to within 6 seconds, yet are about 24 minutes out.

PART - B

1. what is need and importance of mathematics?

It is not very easy to bring out the importance of mathematics education in few words, However, "Few important aspects have been listed below Mathematics education provides.,

- * A good mathematical background with the knowledge of concepts and theories.

- * Ability to apply mathematical concepts and theorems to new situation.

- * A clear understanding of laws of nature.

- * A clear understanding of the culture and development of civilization.

- * Sufficient mathematical skills to meet the demands of daily life.

- * A better understanding of the world around.

- * Ability to make independent decisions in social issues.

- * An essential element of communication.

- * A powerful tool in the hands of the learner.

- * Ability to think alternative methods of solving problems.

2. What is the scope of mathematics :-

Scope of Mathematics :-

Mathematics is all pervasive. So, the demarcation of its scope is a difficult task, if not impossible. It is science of all the science and provides basis to all the disciplines. According to this, the two main aspects of mathematics are,

* Basic Mathematics

* Applied Mathematics,

Basic Mathematics :-

It is also called pure mathematics. The theoretical aspect of mathematics is termed as basic mathematics. It involves systematic and deductive reasoning. It treats only theories and principles without regard to their application to concrete things.

It consists of all those assertions as that if such proposition is true of anything.

* Algebra

* Geometry

* Modern Mathematics

i) Set theory

ii) Topology

iii) Algebraic system

iv) Analysis

v) Combinatorics and Number System

Applied Mathematics:-

It is the application of pure mathematics in developing the various means to solve the human and humanity. It considers those parts of mathematical theories that have certain direct or practical application to objects or actions in the material world.

Principles of Applied Mathematics have been used to investigate phenomena such as heat, light, electricity, sound, mechanics, astronomy etc., some sub-branches of it as follows.

- * Calculatory Science
- * Statistics
- * Numerical Analysis
- * Mathematical Theory of optimization
- * Automation Theory
- * Information Theory
- * Mathematical Aspects of physical theories
 - i) Quantum Mechanics
 - ii) Statistical Mechanics
 - iii) Electromagnetic Theory
 - iv) Theory of relativity
 - v) Dimensional Analysis
 - vi) Riemann Geometry

3. Give brief history of Vedic Mathematics.

Vedic Mathematics:-

* The Vedas are ancient holy texts from India. They can be legitimately characterized as all-encompassing repository of (Hindu) knowledge from cons.

* Vedic Mathematics is the name given to the ancient system of mathematics which was rediscovered from the Vedas between 1911 and 1918 by Sri Bharati Krishna Tirthaji (1884-1960).

* According to his research all of mathematics is based on sixteen Sūtras or word-formulae. For example, 'Vertically and Crosswise' is one of these Sūtras.

* These formula describe the way the mind naturally works and are, therefore, a great help in directing the student to the appropriate method of solution.

* In the Vedic system 'difficult' problems or huge sums can often be solved immediately by the Vedic method.

* These striking and beautiful methods are just a part of a complete system of mathematics, which is far more systematic than the modern 'System'.

* Vedic Mathematics manifests the coherent and unified structure of mathematics and the methods are complementary, direct and easy.

The short cut methods developed by Vedic people to calculate the complex mathematical equations are given below:

- * Ekadhikena Purvena
- * Urdhva - Tiryagbhyam
- * Paravartya Yojayet
- * Shunyam Samasya Samuchaye
- * Puratanapuratanabhyam
- * Yaavadunam
- * Ekanyena Purvena
- * Gunitasamuchyah
- * Gunakasamuchyah

Vedic Number Representation:-

Vedic knowledge is in the form of Slokas or poems in Sanskrit verse. A Number was encoded using consonant groups of the Sanskrit alphabet, and vowels were provided as additional latitude to the author in poetic composition.

4. Explain the contribution of Gauss to mathematics.

Carl Friedrich Gauss (1777-1855), the "Prince of Mathematics", first exhibited his calculative powers when he corrected his father's arithmetic before the age of three.

At the age of 24 years he published what is considered the greatest book of pure mathematics ever - *Disquisitiones Arithmeticae*.

Gauss built the theory of complex numbers essentially from scratch to its modern form, including the notion of "monogenic" functions which are now numerous and in mathematical physics.

The other contributions of Gauss are quite numerous and include the Fundamental theorem of Algebra (that an n^{th} degree polynomial has n roots), the Law of Least Squares, and foundations of Statistics and differential Geometry.

He was the premier number theorist, proving Euler's law of Quadratic Reciprocity.

He also did important work in several areas of physics. Much of Gauss's work wasn't published: unbeknownst to his colleagues it was Gauss who first discovered doubly periodic elliptic functions, non-Euclidean geometry, quaternions, foundations of topology, the "butterfly" procedure for rapid calculation of Fourier series, and even the rudiments of knot theory.

Also in this category is the Fundamental Theorem of Functions of a complex variable (that the line-integral over a closed curve of a monogenic function is zero). He proved this first but let Cauchy take the credit.

5. Elaborate the contribution of Pythagoras for the development of Mathematics.

Contribution to Mathematics:-

Pythagoras is probably most recognized for the theorem about right-angled triangles he made, however, other important contributions to mathematics are described below;

The Square on the Hypotenuse:

This well-known theorem states that the sum of the square of the two sides enclosing the right angle of a right-angled triangles is equal to the square of the long side, called the hypotenuse

Rational and Irrational Numbers:-

The pythagoreans strongly believed that positive integers such as 1, 2, 3, ... had a spiritual implication.

The Numbers were a kind of everlasting certainty, supposed by one's mind. They thought that numbers had a physical existence, and that the universe was made up from them.

To support this idea, they said that musical notes that differed by one octave, or a fifth, could be formed by pipes (like a flute), whose lengths were in the ratios of whole numbers, 1:2 and 2:3 respectively

This discovery followed from applying the pythagorean theorem to the simplest possible right-angled triangle, half a square, a triangle with its two shorter sides both equal to one.

b. why do students get aversion towards mathematics?

* There are several reasons, but my opinion is that most serious reason is that we tell our children that mathematics is difficult and it becomes a self fulfilling prophecy.

* This tends to be more severe in households and communities in lower Socioeconomic homes and communities.

* This seems also to be more severe in families with less formal education.

* Even in families where learning math is encouraged the encouragement may be given in a way that sends the message that math is hard.

* Somehow in our families and our communities we need to send the message that math is inherently no more difficult than other classes.

* They tend to excel in math and science. Not only do their parents push them, but they affirm to them that math and science are not difficult to learn.

* Somehow we need to remove the stigma of math being hard. we need to encourage our children and our students to apply themselves and learn math because math is a powerful tool.

PART - C

1. Mathematics is the science of logical reasoning' - justify.

Mathematics is the science of logical reasoning. Because, this subject deals with both the numbers and logical reasoning. And in Mathematics, each thing has a logical reasoning for using it. Hence, it is known as the science of logical reasoning.

2. write a short note on structure of Mathematics,

In Mathematics, a structure on a set is an additional mathematical object that, in some manner, attaches to that set to endow it with some additional meaning or significance.

3. List out Aryabhatta favorite fields in Mathematics.

* Arithmetic

* Algebra

* plane trigonometry

* Spherical trigonometry

4. Briefly on Euclid's Elements.

The elements is a mathematical treatise consisting of 13 books attributed to the ancient Greek mathematician Euclid in Alexandria. It is a collection of definitions, postulates, propositions, and mathematical proofs of the propositions.

5. characteristics of Mathematics:-

- * Precision and Accuracy
- * A System of logical sequence
- * Applicability
- * Structure of Mathematics