Course - 18(vii) Pedagogy of Mathematics – Part 2

Credit: 4

Essence of the course:

This course is to enable students to specialize in Mathematics and to develop an understanding of the curriculum, linking school knowledge with community life. The course includes reconstruction of Mathematical Knowledge through appropriate pedagogic processes and to communicate meaningfully with children.

Objectives:

At the end of the course, the student teacher will be able to

- acquire the knowledge of competence in teaching Mathematics
- develop clear perception of the Secondary School Mathematics.
- develop awareness of recent trends and principles of construction of Mathematics curriculum.
- know the importance of computers in teaching and learning of Mathematics
- understand the various psychological aspects involved in teaching Mathematics
- know the importance of aesthetic and recreational Mathematics
- develop an understanding of resources of teaching and learning Mathematics.
- help the student teacher for the professional self-development
- enable the student teachers to identify gifted and slow learners in Mathematics and to meet the requirements.
- stimulate to pose and solve meaningful problems and creativity in Mathematics.
- develop insight into individual differences in learning Mathematics to cater to the needs and requirements of students.
- develop skills in construction of appropriate assessment tools for evaluating Mathematics learning

COURSE CONTENT

Unit 1: Revisiting of Content in Mathematics

Definitions, Concepts, Generalizations, Formulae, Laws, Rules, Properties, Axioms, Structures, Constructions, Graphs, Operations, Procedures and Processes, Axioms and Postulates, Theorems and their converse, Propositions, Proofs, Problems etc. in Mathematics Critical analysis of content course of Standard VI to X Mathematics.- Basic concepts in Secondary School Mathematics.

Unit 2: Mathematics Curriculum

Need and importance of Mathematics in School Curriculum - Recent trends in Curriculum Construction - Principles of formulating Mathematics Curriculum - Organization of Syllabus – Topical and Spiral, Logical and Psychological Approaches - Comparison of CBSE and State Board Mathematics syllabi.

Unit 3: Planning and Designing Instruction in Mathematics

Planning Instruction- Need and Importance - Decision Making as the Basis for Planning - Concept of Pedagogic Content Knowledge (PCK) and Components of PCK - Pedagogic Content Knowledge Analysis for selected units in Mathematics at the secondary level in terms of Content, Pre-requisites, Instructional Objectives– Selecting suitable Teaching Methods and

Strategies, Techniques, Models, Learning Activities, Selecting suitable evaluation techniques, Identifying the misconceptions and appropriate remedial measures.

Unit 4: Learning Resources in Mathematics

Mathematics Text Book – Mathematics Library – Mathematics Club and Mathematics Exhibition

Preparation of teaching Aids - Audio-visual Aids and Multimedia Selection and Teaching in Mathematics - Computer applications in Teaching and Learning Mathematics-Uses of ICT in Teaching-learning process.

Unit 5: Psychological foundations of Mathematics Education

Jean Piaget's Cognitive theory, Bruner's Discovery learning, Gagne's eight types of learning and Constructivism - Critical Analysis of Mathematics Curriculum at the secondary level (state board) based on principles and organization of Mathematics curriculum and NCF 2005.

Unit 6: Development of Problem-Solving Ability and Creativity in Mathematics

Meaning – Problems, Problem Solving and Problem Posing – Characteristics of a Good Problem - Problem-solving Strategies and steps in Problem Solving - Gagne's views on Problem Solving - Strategies of Mathematics Problem posing - Divergent Thinking and Creativity in Mathematics - The relation of Creativity to Problem solving and Problem Posing in Mathematics.

Unit 7: Mathematics education for all

Factors influencing the learning of Mathematics-Motivation, Perception, Attitude and Aptitude, Thinking (Divergent and Creativity), etc. - Gifted Children in Mathematics – Meaning, Characteristics and Enrichment programmes, NTSE – Mathematics Olympiad.

Unit 8: Evaluation

Construction and Use of Diagnostic test in Mathematics: Stages, Preparation of Diagnostic Chart (Error Analysis)-Co-operative and collaborative strategies: Learning together, Jigsaw technique – steps -Meaning – Test, Measurement, Assessment and evaluation- Evaluation-General Purposes – Placeof evaluation in Instruction - Types of Evaluation – Placement, Formative, Diagnostic and Summative - Characteristics of a Good Measurement tool - Achievement test – Uses and Construction – Item Analysis - Comprehensive and Continuous Evaluation- Meaning and Functions - Statistics –Central Measures, Measures of Deviation and Graphical Representation

Unit 9: Recreational programme in learning Mathematics

Mathematics Recreational activities and Mathematics Quiz – importance and Organization.

Problems: fear and failure, disappointing curriculum, crude assessment – inadequate teacher preparation- Music Mathematics.

Unit 10: Identification of learning difficulties

Identification of Learning difficulties - Slow Learners in Mathematics – Meaning, Characteristics, Reasons for Slow Learning and learning difficulties: dyslexia, dysgraphia and dyscalculia - remedial measures.

Modes of Transactions:

Lecturing on Theoretical Concepts, Logical Reasoning of Mathematical problems, Analytic and Synthetic Methods of Teaching, Project Method, Tasks and Assignments, seminars.

Learning Activities:

Learning the Content and practicing them appropriately, Oral work, drill, Review and Practicing Pedagogical Aspects for different areas of School Curriculum.

Practicum: Task and Assignment

1. Critically analyze the Mathematics Curriculum at the Secondary Level and prepare a report.
2. Prepare any two improvised teaching aids.
3. prepare the stick album based on the mathematical shapes
4. Search and collect the scrap for Mathematics
5. Prepare a power-point presentation on Mathematical Concepts, Principles and Properties.
6. Prepare remedial measures for any difficulties in learning Mathematics or prepare enrichment programmes for gifted children.
7. Create the collection of mathematical puzzles, riddles for secondary students
8. Collect the mathematical shapes and record it.
9. Search the NET about the mathematical correlation with other subjects
10. Construct any five problems that have multiple right solutions.

Mode of Assessment:

Paper-Pencil Tests, Performance tests, Formal and Informal Testing and Continuous Comprehensive Evaluation.

References:

1. Anice James (2014). Teaching of Mathematics. Hyderabad: Neelkamal Publications Pvt. Ltd.
2. Arul Jothi, Balaji D.L. and NishitMathur (2009). Teaching of Mathematics II. New Delhi: Centrum Press.
3. Bagyanathan, D. (2007). Teaching of Mathematics. Chennai: Tamil Nadu Text Book Society.
4. Bolt, B., & Hobbs, D. (2005). 101 Mathematical projects. New Delhi: Cambridge University
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10. Michael A. Lorber and Walter D. Pierce (1990) Objectives, Methods and Evaluation for Secondary Teaching. New Jersy: Prentice Hall.
11. Nalikar, J. V., &Narlikar, M. (2001). Fun and fundamentals of Mathematics. Hyderabad: Universities Press ltd.
12. NormanE. Gronland (1981). Measurement and Evaluation in Teaching. New York: Macmillan Publishing Co.Inc.
13. Peter N. Ariasian (2000). Assessment in the Classroom: A Concise Approach. New York: McGraw-Hill.
14. Pratap, N. (2008). Teaching of Mathematics. Meerut: R.Lall Books Depot.

15. Schwartz, S. L. (2007). Teaching young children Mathematics. London: Atlantic Publishers & Distributors (P) Ltd.
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17. Sharma, R. A. (2008). Technological foundation of education. Meerut: R.Lall Books Depot.
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19. Singh, M. (2006). Modern teaching of Mathematics. New Delhi: Anmol Publications Pvt. Ltd.
20. Sudhir Kumar and Ratnalikar (2012). Teaching of Mathematics. New Delhi: Anmol Publications Pvt. Ltd.
21. Zubair P.P (2013). Teaching of Mathematics. New Delhi: APH Publishing Corporation.

Web Resources

1. www.infodev.org
2. http://enhancinged.wgbh.org/research/eeeee.html
3. www.classle.net
4. www.ddceutkal.ac.in
5. www.famous-mathematicians.org
6. www.thesecondprinciple.com
7. www.nic.edu
8. www.nctm.org
9. www.arvindguptatoys.com
10. www.slideshare.net
11. www.fpmipa.api.edu
12. www.ricum.edu.rs
13. www.teachingchannel.org
14. www.classroom-aid.com
15. www.ndlrn.edu.au
16. www.bbc.co.uk/learning/subjects/maths.shtml
17. www.primaryresources.co.uk/maths/maths.htm
18. www.mathtutordvd.com

1) Meaning – Problems, Problem Solving and Problem Posing:

- **Problem** is a topic, event or activity, which no memorized or specified rules are known regarding its **solution**.
- **Problem posing** is a crucial component for mathematics discipline and
- **Problem solving** is the situation of elimination of confusion in human mind.

2) Characteristics of a Good Problem

The following are the characteristics of a good problem in mathematics.

- The problem should be real rather than an artificial one.
- It should facilitate the integration of old and new processes.
- The solution of the problem should result in learning new higher order rules.
- The solution of the problem should help in transfer of knowledge.
- The problem should be educationally significant, productive of important and worthwhile learning.
- It should be possible of a solution. The students should be equipped with background information and skills which are prerequisite for solving the given problem.
- It should be related to the sub-unit, the unit and the course.
- It should form the basis for further learning.
- It should be clear and free from ambiguities.
- It should be interesting and challenging.
- It should arouse the curiosity of the students.
- It should occur frequently in life situations.
- It should provide best mental discipline to the students.
- It should have both practical and social values.
- It should be neither too difficult nor too easy for the students.
- It should facilitate realisation of the objectives of teaching mathematics.

8.1.7.5 Reasons for Difficulties in Solving Problems

- Lack of interest and motivation.
- Lack of language clarity in understanding the problem.
- Inability to analyse the problem thoroughly.
- Lack of focus on the key relationships.
- Inability to identify the interrelationship among the given data.
- Lack of fluency in the mental visualisation or diagrammatic representation of the problem.
- Inability to recall and apply appropriate rules and formulae.
- Lack of systematic and orderly written work.
- Lack of skill and practice in solving problems.
- Lack of proficiency in the fundamental arithmetic operations.
- Inadequate knowledge of fundamental mathematical concepts, rules and formulae.
- Difficulty in reading, identifying and using mathematical symbols.

8.1.7.6 Types of Mathematical Problems

Mathematical problems are of *four* types.

- *Puzzle Problem:* These are problems designed for the exercise of ingenuity and patience, as these problems create some bewilderment or perplexity in the individual who faces it. Sometimes people solve them as a leisure time activity merely for the sake of joy and pleasure that they derive. However, such problems preserve the curiosity of the student and he feels joy in solving them.
- *Catch Problem:* These problems display a jugglery of words. Such problems check the mental alertness of the students, but have little bearing on training mental faculties.
- *Real Problems:* These problems are directly related to the real life experiences of the students. They emerge from the real life situations. The solution of such problems help the students in facing future life problems with ease and confidence. The solution of such problems stimulates the curiosity and help in training the mental faculties of the children.
- *Unreal Problems:* Problems which are beyond the purview of real life situations are called unreal problems. Such problems give false information to the students.

3) Problem-solving Strategies and steps in Problem Solving - Gagne's views on Problem Solving

8.1.7 Problem-Solving Method

The problem-solving method is one which involves the use of the process of problem-solving or reflective thinking or reasoning. Problem-solving method, as the name indicates, begins with the statement of a problem that challenges the students to find a solution. The problem centres around the subject matter under study and requires the use of information and skills available to the students. In the process of solving the problem the students may be required to gather data, analyse and interpret the information, to arrive at a solution to the problems.

Definitions of Problem-Solving

Problem solving presupposes the existence of a problem in the teaching-learning situation. A problem is an obstruction of some sort to the attainment of an objective, a sort of difficulty which does not enable the individual to reach a goal easily.

"A problem occurs in a situation in which a felt difficulty to act is realised. It is a difficulty that is clearly present and recognised by the thinker. It may be a purely mental difficulty or it may be physical and involve the manipulation of data. The distinguishing thing about a problem, however, is that it impresses the individual who meets it as needing a solution. He recognises it as a challenge". – ***Yokam and Simpson***

"Problem solving is a planned attack upon a difficulty or perplexity for the purpose of finding a satisfactory solution". – ***Risk, T.M***

"Problem solving is an educational device whereby the teacher and the pupils attempt in a conscious, planned, purposeful manner to arrive at an explanation or solution to some educationally significant difficulty".

– ***James Ross***

From the above definitions, problem solving involves the following.

- a goal to be reached
- a felt difficulty to reach the goal
- challenging the felt difficulty through conscious, planned and purposeful attack
- reaching the goal or arriving at a satisfactory solution to the problem at hand

Therefore, as used in teaching-learning situation, problem-solving is a method in which the felt difficulty to act in an educational situation is realised and then an attempt is made in a conscious and purposeful way to find its solution.

Main Objectives of Problem-Solving Method

The main objective of problem-solving method is to stimulate the reflective and creative thinking of the students. It involves the thought process that results from a doubt, a perplexity or a problem. The approach leads to the formulation of generalisations that are useful in future situations involving the solution of similar problems. The solution of a problem, whatever be its nature, practical or informational involves the process of reflective thinking.

8.1.7.1 Steps in Problem-Solving

Problem-solving follows definite and specific steps.

Identifying and defining the problem

The problem arises out of a felt need and out of existing student activities and environment activities. The students should be able to identify and clearly define the problem. The problem that has been identified, should be interesting, challenging and motivating for the students to participate in exploring.

Analysing the problem

The problem should be carefully analysed as to what is given and what is to be found out. Given facts must be identified and expressed, if necessary in symbolic form. The relationships are to be clearly stated. Relations that are not explicitly stated may be supplied by the students.

Formulating tentative hypothesis

The focus at this stage is on hypothesising – searching for a tentative solution to the problem. Analysis of the given data, and analysis of interrelationships among the given facts help the students in formulating hypothesis or educated guesses as the solution to the problem at hand.

Testing the hypothesis

Appropriate methods should be selected to test the validity of the tentative hypothesis as a solution to the problem. If it is not proved to be the solution, the students are asked to formulate alternate hypothesis and proceed.

Checking the result or Verification of the result

At this step the students are asked to determine their results and substantiate the expected solution. The students should be able to make generalisations and apply it to their daily life.

Gagne's views on Problem Solving

Problem Solving is an extention of rule learning .Problem solving requires an individual to discover a combination of previously learned rules to apply to solve a novel problem. Problem solving combines two or more rules to produce a new capability, resulting in the formation of a higher order rule. Higher order rules are learning strategies which enable individuals to solve other problems of a similar type and such higher-order rules often result from the learner's thinking in a problem-solving situation.

Problem solving becomes associated with both intellectual skills and cognitive strategies.

Condition for Problem Solving

i) The rules must be previously learned by the learner

ii) The learner should have verbal ability and language skill to read and understand the problem.

iii) The learner should be able to recall and apply the appropriate rules

iv) The learner must use cognitive strategies to solve the problem.

8.1.7.2 Approaches and Techniques to Problem-Solving

Problem-solving advocates the following approaches

- *Analytic and synthetic methods.*
- *Inductive and deductive methods.*

(For details refer to earlier parts of this chapter)

- *Method of analogies*

  In analogy, problems are solved by comparing them with similar problems that have been solved before. Thus the method of solution becomes explicit and clear.

- *Restatement Method*

  Problem solving becomes easier if the student is able to redefine the given problem using his own language and symbols. This approach is known as restatement method.

- *Method of Dependencies*

  In this method, the problem is solved by focussing on mutually dependent components in the problem. The analysis of the problem into its constituent elements throws light on the mutually dependent elements in the problem. The interrelationships among the elements can be made use of for reaching the correct solution of the problem.

- *Graphic Method*

  In this method, the problem is represented using diagrams and figures. The graphic representation aids the students in determining fundamental relationships that exist among the given data and to look for further details and relationships necessary for solving the given problem. This method is very helpful in proving theorems, solving riders, problems relating to mensuration. Pythagoras theorem, set theory, functions and relations etc.

8.1.7.3 Teacher's Role in Problem-Solving Method

The teacher plays a significant role in problem solving method. The teacher's role is to:

- ensure an atmosphere of freedom in the class.
- create the problem situation.
- assist the students in accepting, defining and stating the problem.
- help the students in analysing the problem and in breaking up the problem into simple units.
- help the students keep their attention focussed on the main problem all the time.
- guide the students in locating relevant source materials.
- encourage the students in seeking important relationships in the data.
- help the students develop an attitude of open mindedness and critical enquiry
- exhibit spirit of enquiry and discovery

8.1.7.7 Merits of Problems-Solving Method

- Problem-solving provides a real life like experience to the children.
- It develops in pupils good habits of planning, thinking, reasoning and independent work.
- It develops initiative and self-responsibility among the students.
- It takes into account individual differences.
- It helps the students to develop reflective thinking.
- It helps the students to approach future problems with confidence.
- It builds a mental attitude for effective learning based on critical thinking
- It helps the children develop mental traits of open-mindedness and tolerance as the children see many sides to a problem and listen to many points of view.

8.1.7.8 Demerits of Problem-Solving Method

- Not all students are problem solvers.
- The problem solving method becomes monotonous if used too frequently.
- It is time consuming and consequently it is not possible to cover the syllabus on time.
- The success of this method depends upon mathematics teachers who are well versed in critical thinking and reflective thinking. Not all mathematics teachers are well versed in these types of thinking.
- Reference and resource materials may be difficult to come by.
- Only a skilled and resourceful teacher will be able to make an effective use of this method.
- All topics in mathematics cannot be taught through this method.
- Textbooks are not available according to this method.
- Lack of interest and motivation on the part of the students can spoil the effectiveness of this method.

Conclusions

Problem-solving method can be an effective method for teaching mathematics in the hands of an able and resourceful teacher of mathematics.

4) Strategies of Mathematics Problem posing

Problem posing is closely associated with the problem solving method. Problem posing involves generating new problems and questions to explore about a given situation, as well as reformulating a problem during the course of solving the problem related to it. Teachers can help to developing this habit by understanding the children's thinking processes and developing these processes using generative questions. The problem posing method involves developing problem posing as an instructional intervention to improve problem solving skills and to improve disposition towards solving. Problem posing is an indicator of learning that takes place.

When we encourage children to be problem posers, we are inviting them to do what mathematicians do — that is, to look closely, seek patterns, offer conjectures, and set out on paths that are not clearly marked. In the process of their investigations, mathematicians also develop attitudes about learning, such as perseverance, willingness to revise their thinking, and appreciation for the value of risk taking.

Let us, as an example, consider the statement *4X 5 = 20.*

The first step of problem posing is to look closely or observe the statement critically. In the above statement following are some of the observations we can make.

There are two multipliers.

The two multipliers are two consecutive natural numbers.

One of the multipliers is even and other is odd.

One is a multiple of 2 and other is multiple of 5.

The product is 4 more than a square number (16) and 5 less than another square number (25).

The multipliers are consecutive counting numbers.

The difference between the multipliers is 1.

After making the observation on the statement *4X 5 = 20,* what possible problems can be posed? Some problems posed here as exemplars:

1. Do we always get an even product when multiplying an odd number by an even number?

2. What do we find if we multiply an odd number by an odd number? An even number by an even number? What if we multiplied three odd numbers or three even numbers?

3. What if we continued to multiply by multiples of 2 and multiples of 5? What patterns might we see?

4. What if we tried using multipliers that are the same to make 20? Is this result possible? What products are possible using multipliers that are the same?

5. What if we tried adding two numbers to equal 20? How many ways could we do so? What do we notice about odd and even numbers when adding to make 20?

6. Why, when we add an odd and an even number, do we get an odd number, but when we multiply an odd number by an even number we get an even number?

What are the benefits of problem posing for learning?

It develops the spirit of inquiry. The more we observe, the more we want to find out.

It leads the learner into unknown territory.

It requires and promotes reflective thinking especially during posing the problems.

It supports learners in asking the perennial question that mathematicians pose: Is this always true? That is, did this relationship occur fortuitously, or does a pattern lurk behind these numbers?

The other benefit is that problem posing involves searching for patterns.

Uncovering patterns is certainly joyful, but even more rewarding is discovering why those patterns are occurring. Let's see what we notice about our original problem: 4× 5 = 20.We see that 20 is 4 away from the nearest smaller square, 16, and is 5 away from the nearest larger square, 25. Why? We notice that 4 × 5 is 4 x (4+1) or (5-1) x 5

MATHEMATICAL PROBLEM POSING STRATEGIES

Mathematics teachers might use one or more strategies to formulate new problems or encourage their students in mathematics classes to be good problem posers as well as a good problem solvers. Strategies could be used depending on the most suitable conditions (mathematics content, students' levels, learning outcomes and mathematical thinking types). Problem posing situations are classified as free, semi-structured or structured situations.

Free Problem Posing Situations

Situations from daily life (in or outside school) can help a student to generate some questions leading him/her to construct a problem. Students are asked to pose a problem to encourage them to "make up a simple or difficult problem" or "construct a problem suitable for a mathematics competition

(or a test)" or "make up a problem you like." It is more useful if the teacher tries to relate the real life situations to the mathematics content being taught and to ask students to pose new problems. This will be more effective in developing students' mathematical thinking. Problem posing situations might take these types: every day life situation, free problem posing, problems they like, problems for a mathematics competition, problems written for a friend and problems generated for fun.

Semi-Structured Problem Posing Situations

Students are given an open-ended situation and are invited to explore it using knowledge, skills, concepts and relationships from their previous mathematical experiences and this can take the following forms:

Open-ended problems (i.e. mathematical investigation).

Problems similar to given problems.

Problems with similar situations.

Problems related to specific theorems.

Problems derived from given pictures.

Word problems.

This strategy was developed with student teachers as the following (Abu-Elwan, 1999):

1) A semi-structured situation from a student's daily life was presented to all students.
2) Students were asked to complete the situations using their perspective to be able to pose problems from that formed situation.

Students can generate problems by omitting the questions from given situations.

Structured Problem Posing Situation

Any mathematical problem consists of known (given) and unknown (required) data. The teacher can simply change the known and pose a new problem, or keep the data and change the required. Brown and Walter (1990, 1993) designed an instructional problem formulating approach based on the posing of new problems from already solved problems, but they have also recommended varying the conditions or goals of given problems.

This reformulation approach appears to be the most effective method for introducing structured problem posing activities in mathematics classrooms.

In order to create teaching/learning situations that provide a good problem posing situations, Lowrie (1999) recommended the mathematics teacher to:

1) encourage students to pose problems for friends who are at or near their own level until they become more competent in generating problems;
2) ensure that students work cooperatively in solving the problems so that the problem generator gains feedback on the appropriateness of the problems they have designed;
3) ask individuals to indicate the type of understanding and strategies the problem solver will need to use in order to solve the problem successfully before a friend generates a solution;
4) encourage problem solving teams to discuss, with one another, the extent to which they found problems to be difficult, confusing, motivating or challenging;
5) pProvide opportunities for less able students to work cooperatively with a peer who challenged the individual to engage in mathematics at a higher level than they were accustomed;
6) challenge students to move beyond traditional word problems by designing problems that are open ended and associated with real life experiences; and
7) encourage students to use technology (calculators, CDs, computers) in developing their mathematical thinking skills, so they can use this technology to generate new mathematical situations.

5) Divergent Thinking and Creativity in Mathematics

6.7 Divergent Thinking in Mathematics

Creative production is often characterized by the divergent nature of human thought and action. Divergence is usually indicated by the ability to generate many, or more complex or complicated, ideas from one idea or from simple ideas or triggers. Divergent Thinking is thinking outwards instead of inward. It is the ability to develop original and unique ideas and then come up with a problem solution or achieve an objective.

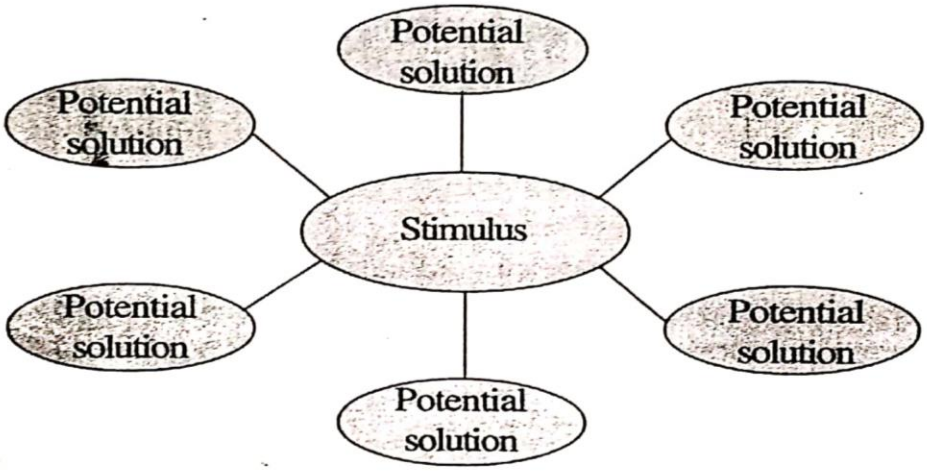

Fig.6.1: Divergent Thinking

Traditionally the eight elements below are ones commonly thought of as inherent elements of divergent thinking.

- **Fluency** - The ability to generate a *number of ideas* so that there is an increase of possible solutions or related products.
- **Flexibility** - The ability to produce different *categories or perceptions* whereby there are a *variety* of different ideas about the same problem or thing.
- **Elaboration** - The ability to *add to, embellish, or build off of* an idea or product.
- **Originality** - The ability to *create fresh, unique, unusual, totally new, or extremely different* ideas or products
- **Complexity** - The ability to conceptualize *difficult, intricate, many layered or multifaceted* ideas or products.

- **Risk-taking** - The willingness to be *courageous, adventuresome, daring* – trying new things or taking risks in order to stand apart.
- **Imagination** - The ability to *dream up, invent,* or to see, to think, to conceptualize new ideas or products to be ingenious.
- **Curiosity** - The trait of exhibiting *probing behaviors,* asking and posing questions, searching, being able to look deeper into ideas, and the wanting to know more about something.

6.8 Creative Thinking in Mathematics

Creativity is an attitude of mind which is encouraged by openness of thinking, willingness to work with conflicting ideas and not to have the solutions immediately, eagerness to learn, an appreciation of the working of the unconscious and a preparedness to play with an imagination and by a readiness to stand back and question the obvious. By providing the environment where the students have full freedom of thought, able to recognize their experiences independent of external restraint is conducive for promoting creative thinking. Creativity in teaching breeds creativity in learning. Creativity is defined as involving the ability to produce work that is both novel (original, unexpected) and appropriate. Creative thinking enhances problem solving ability and it provides as a tool of learning process. There are different stages in creative thinking such a separation, incubation, illumination and verification.

What is common in a lot of school mathematics problems is that they are supposed to have only one solution. Pehkonen (1995) defined this type of problem as a closed problem. He suggested that these particular problems, which do not allow divergent thinking, are not able to enhance the quality of education, even after the introduction of new approaches. If the goal of mathematics teaching is to realize an individual's potential of mathematical creativity, it is necessary to break the habit of "knowledge delivery" from the teacher to the student, which is the conventional teacher-oriented teaching method. Mathematics teaching should be focused on the development of creative thinking where students are free to try their own original possible solutions. It means avoiding the traditional teaching method that emphasizes 'convergent thinking,' in which a student memorizes existing mathematical rules and theorems and then applies them to problems with great adroitness in order to find one exclusive solution.

Since these closed problems do not encourage students to adopt divergent thinking and reasoning, it is necessary to introduce new contexts

that allow the students to respond positively and participate actively in the learning process. They are

6.8.1 Brainstorming

It is a creative problem solving technique which is used to record maximum possible number of ideas on a defined subject. This brainstorming sequence includes sensing the problem, fact finding, ideation, and evaluation of ideas and planning for implementation.

6.8.2 Self-directed Learning

It is a systematic process in which the students take up the responsibility in collaboration with others for diagnosing their own learning needs formulating learning objectives planning and engaging in a sequence of learning experience.

6.8.3 Discovery Learning

This helps the learner to achieve his own understanding of mathematics by successfully solving a carefully structured sequence of problems. Here the student discovers the knowledge independently by experimentation in exploration instead of being directly presented with a content to be learnt.

6.8.4 Guided Discovery

Here the students are not acquainted with facts rather they are made to investigate or discover the facts. The teacher only provides suitable direction to produce independent solutions to problems.

6.8.5 Encouraging Intrinsic Motivation

Creativity is developed through higher levels of intrinsic motivation. The strategies which may influence intrinsic motivation are challenge, freedom, resources and supervisory encouragement.

6.8.6 No Over Controlling

Instead of dictating the activities they should engage in, the teachers let the students select their interests and support their inclinations.

6.8.7 Fostering Flexible and Playful Thinking

Creative thinkers are flexible and play with the problems which give rise to a paradox. The considerations here are

- Being open to alternate solutions

- Practicing creativity by regularly engaging the students in activities that encourage flexible thinking
- Using multiple senses when seeking solutions by thinking in terms of the five basic senses, the kinaesthetic sense and visualizing how the solution must look.
- Playing the roles of explorer, artist, judge and lawyer

6.8.8 Questioning

The questions that elicit many answers can be put whole teaching mathematics at any level in the classroom. The students should be confronted with open ended and divergent questions.

6.8.9 Encouraging Lateral Thinking

Lateral thinking requires the students to deviate from the conventional style of vertical thinking. It involves the flexibility of being able to change ones perception of the situation to insight full situations.

6) The relation of Creativity to Problem solving and Problem Posing in Mathematics.

1. Mathematics creativity refers to discovering new connections as a result of formal changes to things that already exists.
2. Asking original questions to solve problems and presenting solutions from various viewpoints have also been referred to as mathematics creativity.
3. Mathematics creativity as two different products they are cognitive processes and result-oriented endeavors.
4. Creativity is in the nature of problem posing that is creating a problem is a creative activity.
5. It can be determined by an original solution to a problem that no one has solved before.
6. The three criteria for math creativity are fluency, flexibility and originality.

7. Problem solving is the process of understanding the problem which is the first stage of creative process.
8. Problem posing is the process of examining the problems, analyzing them and writing up using their own words.
9. It is proved that geometry has been a proper field for showing more than one way to solve a problem and then they assessed their subject's geometry knowledge and creativity using a geometry problem.
10. For example, if a student reaches a solution using a different method of others, he or she has a higher level of creativity than do others. In general problem posing, mathematics creativity and problem solving have common characteristics.

6.6 Factors Influencing the Learning of Mathematics

6.6.1 Motivation

Motivation is a potential to direct behaviour that is built into the system that controls emotion. This potential may be manifested in cognition, emotion and/or behaviour.

Three basic factors are related to motivation.

1. Need 2. Drive 3. Motive

Need refers to lack of something, absence of something, non availability of something which is essential or desirable, whereas drive is the force which impels the individual to initiate to action to meet his/her needs.

Motive refers to a thought or feeling which generates a drive in the individual and this drive in turn will impel the individual towards action to meet his need.

Hence motivation can be defined as the process of installing appropriate motives, which are likely to facilitate corresponding drives, which in turn are likely to initiate action towards meeting the needs.

The factors affecting motivation while learning mathematics are:

- level of aspiration
- reward and punishment,
- social motives, and
- competition

How to motivate the students to learn mathematics?

The students can be motivated to learn mathematics by emphasizing the need for achievement.Training in behaviour on

- how to take moderate risks
- how to develop self confidence in one's ability to solve long range problems
- how to be challenged by difficult tasks
- how to look for feedback in one's long range performance

Would help the individuals develop a need for high achievement.

To develop motivational functions to revitalize the students for learning of mathematics, mathematics teachers should

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- guard against monotony and boredom,
- increase the general level of alertness and responsiveness
- be sensitive to individual differences in learning.
- be resourceful in classroom teaching,
- maintain a conducive classroom climate for effective learning,
- employ a variety of teaching techniques ensuring student involvement and attention.
- set before the students specific learning objectives
- replace their long time goals by immediate goals
- encourage students to improve the self concept

Finally teacher's personality plays a vital role in motivation. If the teacher is impressive, pleasing with effective communication skills he/she can easily motivate his/her students.

Basic principles of motivation those are applicable to learning of mathematics are,

❖ *The environment can be used to focus the student's attention on what needs to be learned.*

Teachers who create warm and accepting yet business-like atmospheres will promote persistent effort and favorable attitudes toward learning. Interesting visual aids motivate learners by capturing their attention and curiosity.

❖ *Incentives motivate learning.*

Incentives include privileges and receiving praise from the teacher. In a learning situation, self-motivation without rewards will not succeed. Students must find satisfaction in learning based on the understanding that the goals are useful to them or, less commonly, based on the pure enjoyment of exploring new things.

❖ *Internal motivation is long lasting and more self-directive than is external motivation, which must be repeatedly reinforced by praise or concrete rewards.*

Some students have little capacity for internal motivation and must be guided and reinforced constantly. The use of incentives is based on the principle that learning occurs more effectively when the student experiences feelings of satisfaction. Caution should be exercised in using external rewards when they are not absolutely necessary.

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Learning is most effective when the learner is ready to learn, that is, when one wants to know something.

Sometimes the student's readiness to learn comes with time, and the teacher's role is to encourage its development. If a student is not *ready to learn,* he or she may not be reliable in following instructions and therefore must be supervised and have the instructions repeated.

- *Motivation is enhanced by the way in which the instructional material is organized.*

  In general, the best organized material makes the information meaningful to the learner. One method of organization includes relating new tasks to those already known. Other ways to relay meaning are to determine whether the persons being taught understand the final outcome desired and instruct them to compare and contrast ideas.

None of the techniques will produce sustained motivation unless the goals are realistic for the learner. The basic learning principle involved is that *success is more predictably motivating than is failure.*

Students develop interest, pay attention, highly-motivated to learn mathematics, and develop a positive attitude towards mathematics provided students are assisted in defining goals which may increase the probability that they will understand them and want to reach them. To identify realistic goals, teachers must be skilled in assessing students readiness or a students' progress toward goals.

- *Because learning requires change in beliefs and behavior, it normally produces a mild level of anxiety.*

  This is useful in motivating the individual. However, severe anxiety is incapacitating. A high degree of stress is inherent in some educational situations. If anxiety is severe, the individual's perception of what is going on around him or her is limited. Teachers must be able to identify anxiety and understand its effect on learning. They also have a responsibility to avoid causing severe anxiety in learners by setting ambiguous or unrealistically high goals for them.

- *It is important to help each student set goals and to provide informative feedback regarding progress toward the goals.*

  Setting a goal demonstrates an intention to achieve and activates learning from one day to the next. It also directs the student's activities toward the goal and offers an opportunity to experience success.

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- *Both affiliation and approval are strong motivators.*

  People seek others with whom to compare their abilities, opinions, and emotions. Affiliation can also result in direct anxiety reduction by the social acceptance and the mere presence of others. However, these motivators can also lead to conformity, competition, and other behaviors that may seem as negative.

- *Many behaviors result from a combination of motives.*

Motivational factors and strategies

Motivation is so necessary for learning that strategies should be planned to organize a continuous and interactive motivational dynamics for maximum effectiveness. The general principles of motivation are interrelated. A single teaching action can use many of them simultaneously.

Time	*Motivational Factors*	*Motivational Strategies*
Beginning: When learner starts learning	**Attitudes:** attitudes toward the environment, teacher, subject matter, and self **Needs:** the basic need within the learner at the time of learning	Make the conditions that surround the subject positive. – Positively confront the possibly erroneous beliefs, expectations, and assumptions that may underlie a negative learner attitude. – Reduce or remove components of the learning environment that lead to failure or fear. – Plan activities to allow learners to meet esteem needs
During: When learner is involved in main content of the learning process.	**Stimulation:** the stimulation processes affecting learner during the learning experience. **Affect:** the emotional experience of the learner while learning.	– Change style and content of the learning activity. – Make learner reaction and involvement essential parts of the learning process, that is, problem solving, role playing, stimulation.

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Time	*Motivational Factors*	*Motivational Strategies*
		– Use learner concerns to organize content and to develop themes and teaching procedures. – Use a group cooperation goal to maximize learner involvement and sharing.
Ending: When learner is completing the learning process.	**Competence**: The competence value for the learner that is a result of the learning behaviors.	– Provide consistent feedback regarding mastery of learning.
	Reinforcement: The reinforcement value attached to the learning experience, for the learner	– Acknowledge and affirm the learners' responsibility in completing the learning task. – When learning has natural consequences, allow them to be congruently evident. – Provide artificial reinforcement when it contributes to successful learning, and provide closure with a positive ending.

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6.6.3 Perception

Learning refers to a relatively permanent change in behavior that is a result of past experience or practice. Perception is a general term referring to the awareness of objects, qualities, or events stimulating the sense organs; it also refers to a personal experience of the world. Perception is a meaningful predation of a sensation. A large part of our learning is accompanied by perception.

The meaning we attach to what we sense is called perception. Perception is an organizing process. In perception we organize, integrate and recognize the various patterns of stimuli. Learning of mathematics and Perception are intertwined. Perception in learning of mathematics is basically

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a pattern-recognition process coupled with some functional consequences for the system which performs such pattern-recognition.

Without perception the higher level cognitive process such as imagination, thinking, reasoning and problem solving will not function. Perception furnishes the experience that promotes understanding and reflective thinking. Perceptions are modified by the activities themselves by trial and error methods. It is enhanced by needs motivations and value systems. Hence, field trips, laboratory work are important supplements to teaching and learning of mathematics.

Knowledge of factors which may affect the perpetual process of learning mathematics is very important to the mathematics teacher because perception is the basis of all learning. There are several factors that may affect the students' ability to perceive. They are

1. basic needs
2. goals and values
3. self concept
4. time and opportunity
5. element of threat and
6. insight

True learning requires an understanding of how each of these factors may affect all of the others and the knowledge of how a change in any one of them may affect all the others. It is a major responsibility of the mathematics teacher to organize demonstrations and explanations and direct practice so that the student has better opportunities to understand the interrelationship of the many kinds of experiences that have been perceived. Pointing out the relationships as they occur, providing a secure and non threatening environment to learn mathematical concepts and helping the student acquire and maintain a favourable self concept are key steps which may foster the ability to perceive the mathematical patterns.

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6.6.5 Attitude

Attitudes are defined as predispositions to react in a certain way to an object or experience. They are learned and they influence our actions. Attitudes are viewed to consist of the three components: affect, cognition, and behaviour. Attitudes consist of evaluative dimensions based on the three components in combination or on their own. The affective component consists of physiological reactions and verbal statements of feelings; cognitive evaluations can be perceptual responses or verbal statements of

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beliefs; and overt actions and verbal reports of behaviour represent the behavioural component.

As the modern society has become increasingly dependent upon technology, science and research, mathematics has become critical in the preparation of students for future careers and progress of the nation. Research shows that attitude towards mathematics is extremely important in the achievement and participation of students in mathematics.

Recognizing the importance of attitudes, there is an increasing awareness of the need to examine attitudes and consider possible methods of involvement. The development of a positive attitude towards the subject matter is probably one of the most prevalent goals.

Attitudes influence success and persistence in the study of mathematics.

Students who have positive feelings about mathematics exert more effort, spend more time on mathematics tasks, and are more effective learners than students with poor attitudes.

6.6.6 Aptitude

Human efficiency is not as easily defined as that of a machine and it is not easily measured. Generally speaking, however, we consider one person as more efficient than another, if he accomplishes more in the same time. The working efficiency of an individual varies with a number of factors, the most important of which are (i) his aptitude for the task involved, (ii) adequacy of his training for the job and (iii) his mindset and conditions of work. Like intelligence, aptitude is measured in terms of individual differences. In a given type of work, there are those who learn rapidly and achieve a high level of skill and those who are slow to learn and whose achievement is low. The former are said to have good aptitude for the work in question.

An aptitude is a set of characteristics regarded as symptomatic of an individual's ability to acquire with competence (usually specified) knowledge, skill or set of responses such as the ability to speak a language, to produce music etc. In referring to a person's aptitude for mathematics, or art or carpentry we are looking to the future. His aptitude is however, a present condition, a pattern of traits deemed to be indicative of his potentialities for the future. However, in this definition there is no indication as to whether the 'condition' or 'set of characteristics' is acquired or inborn. We want the facts about an individual's aptitude as they are at present; characteristics now indicative of his future potentialities, whether he was born that way or

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acquired certain enduring dispositions in his earlier childhood, or matured under circumstances which have radically altered his original capacities. An individual's potentialities at the period of development are quite certainly the product of interaction between conditions both innate and environmental. Hence aptitudes are probably inherited because they cannot appear unless the environment is conducive, i.e. unless the opportunity is provided. Very often some training, often a good deal of it, is necessary too, before an aptitude reveals itself in performance. While appraising an individual's aptitude, we must take him as he is, and not as he might have been.

Aptitude, moreover, connotes more than *potential ability* in performance, it implies *fitness* and *suitability* for the activities in question. One does not deem self-suited for a work for which he has distaste or an occupation that offers no challenge to one's interest. When appraising aptitude we are on the alert for symptoms of ability to acquire genuine absorption in the work as well as satisfactory level of competence. A person with an aptitude for a job will show enthusiasm and necessary drive towards the job in hand.

Aptitude is differentiated from skill and proficiency. Skill means the ability to perform a given act with ease and precision. Proficiency has much the same meaning except that it is more comprehensive, for it includes not only skill in certain types of motor and manual activities, but also in other type of activities as shown by the extent of one's competence in language, book keeping, mathematics etc. On the other hand, when we speak of a person's aptitude for a given task, we mean the capacity to acquire proficiency under appropriate conditions, that is, his potentialities at present, as revealed by his performance on the selected tests that have predictive value. Such tests are known as *aptitude tests.*

Mathematics Aptitude consists of the following tests:

- *Visual Matching*

  This measures the student's ability to demonstrate visual discrimination.

- *Analysis-Synthesis*

  This measures the student's ability to use learning, reasoning, and generalizing skills in solving logic puzzles.

- *Oral Vocabulary*

  This measures the student's ability to provide synonyms and antonyms for a list of presented words.

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- *Concept Formation*

  This measures the student's ability to utilize nonverbal problem solving strategies and categorical reasoning.

6.7 Divergent Thinking in Mathematics

Creative production is often characterized by the divergent nature of human thought and action. Divergence is usually indicated by the ability to generate many, or more complex or complicated, ideas from one idea or from simple ideas or triggers. Divergent Thinking is thinking outwards instead of inward. It is the ability to develop original and unique ideas and then come up with a problem solution or achieve an objective.

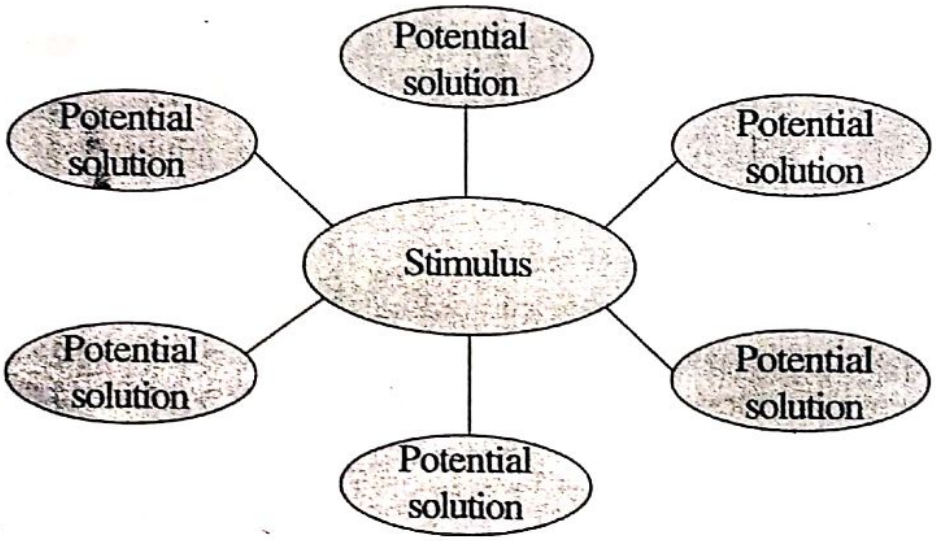

Fig.6.1: Divergent Thinking

Traditionally the eight elements below are ones commonly thought of as inherent elements of divergent thinking.

- **Fluency** - The ability to generate a *number of ideas* so that there is an increase of possible solutions or related products.
- **Flexibility** - The ability to produce different *categories or perceptions* whereby there are a *variety* of different ideas about the same problem or thing.
- **Elaboration** - The ability to *add to, embellish, or build off of* an idea or product.
- **Originality** - The ability to *create fresh, unique, unusual, totally new, or extremely different* ideas or products
- **Complexity** - The ability to conceptualize *difficult, intricate, many layered or multifaceted* ideas or products.

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- **Risk-taking** - The willingness to be *courageous, adventuresome, daring* – trying new things or taking risks in order to stand apart.
- **Imagination** - The ability to *dream up, invent,* or to see, to think, to conceptualize new ideas or products to be ingenious.
- **Curiosity** - The trait of exhibiting *probing behaviors,* asking and posing questions, searching, being able to look deeper into ideas, and the wanting to know more about something.

6.8 Creative Thinking in Mathematics

Creativity is an attitude of mind which is encouraged by openness of thinking, willingness to work with conflicting ideas and not to have the solutions immediately, eagerness to learn, an appreciation of the working of the unconscious and a preparedness to play with an imagination and by a readiness to stand back and question the obvious. By providing the environment where the students have full freedom of thought, able to recognize their experiences independent of external restraint is conducive for promoting creative thinking. Creativity in teaching breeds creativity in learning. Creativity is defined as involving the ability to produce work that is both novel (original, unexpected) and appropriate. Creative thinking enhances problem solving ability and it provides as a tool of learning process. There are different stages in creative thinking such a separation, incubation, illumination and verification.

What is common in a lot of school mathematics problems is that they are supposed to have only one solution. Pehkonen (1995) defined this type of problem as a closed problem. He suggested that these particular problems, which do not allow divergent thinking, are not able to enhance the quality of education, even after the introduction of new approaches. If the goal of mathematics teaching is to realize an individual's potential of mathematical creativity, it is necessary to break the habit of "knowledge delivery" from the teacher to the student, which is the conventional teacher-oriented teaching method. Mathematics teaching should be focused on the development of creative thinking where students are free to try their own original possible solutions. It means avoiding the traditional teaching method that emphasizes 'convergent thinking,' in which a student memorizes existing mathematical rules and theorems and then applies them to problems with great adroitness in order to find one exclusive solution.

Since these closed problems do not encourage students to adopt divergent thinking and reasoning, it is necessary to introduce new contexts

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that allow the students to respond positively and participate actively in the learning process. They are

6.8.1 Brainstorming

It is a creative problem solving technique which is used to record maximum possible number of ideas on a defined subject. This brainstorming sequence includes sensing the problem, fact finding, ideation, and evaluation of ideas and planning for implementation.

6.8.2 Self-directed Learning

It is a systematic process in which the students take up the responsibility in collaboration with others for diagnosing their own learning needs formulating learning objectives planning and engaging in a sequence of learning experience.

6.8.3 Discovery Learning

This helps the learner to achieve his own understanding of mathematics by successfully solving a carefully structured sequence of problems. Here the student discovers the knowledge independently by experimentation in exploration instead of being directly presented with a content to be learnt.

6.8.4 Guided Discovery

Here the students are not acquainted with facts rather they are made to investigate or discover the facts. The teacher only provides suitable direction to produce independent solutions to problems.

6.8.5 Encouraging Intrinsic Motivation

Creativity is developed through higher levels of intrinsic motivation. The strategies which may influence intrinsic motivation are challenge, freedom, resources and supervisory encouragement.

6.8.6 No Over Controlling

Instead of dictating the activities they should engage in, the teachers let the students select their interests and support their inclinations.

6.8.7 Fostering Flexible and Playful Thinking

Creative thinkers are flexible and play with the problems which give rise to a paradox. The considerations here are

- Being open to alternate solutions

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- Practicing creativity by regularly engaging the students in activities that encourage flexible thinking
- Using multiple senses when seeking solutions by thinking in terms of the five basic senses, the kinaesthetic sense and visualizing how the solution must look.
- Playing the roles of explorer, artist, judge and lawyer

6.8.8 Questioning

The questions that elicit many answers can be put whole teaching mathematics at any level in the classroom. The students should be confronted with open ended and divergent questions.

6.8.9 Encouraging Lateral Thinking

Lateral thinking requires the students to deviate from the conventional style of vertical thinking. It involves the flexibility of being able to change ones perception of the situation to insight full situations.

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7.4 Identification of the Gifted and Enrichment Programmes for the Gifted

The identification of the mathematically gifted is as important as nurturing their mental abilities and skills to acquire a high level mathematical thinking and reasoning.

The unique characteristics exhibited by the gifted students will help the teacher in identifying them. However, the teacher has to carefully follow their academic and other performance consistently for a long time before he identifies them as gifted.

The following points should be taken into account while classifying the students as mathematically gifted or mathematically weak.

- Opinion of other subject teachers.
- Students' score on mathematics aptitude test, mathematics achievements test and intelligence test.
- Students' past performance in mathematics in the previous classes.

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- Students' score on inventories like Interest in mathematics, Attitude towards mathematics.
- The report of a properly planned interview.

7.4.1 Characteristics of the Mathematically Gifted

The Secondary School Curriculum Committee gives the following general and special characteristics which mark the mathematically gifted.

General characteristics

- Has excellent memory, good vocabulary, broad attention span, and high reading ability.
- Makes associations readily and retains them indefinitely.
- Recognizes similarities and differences quickly.
- Has a relatively mature sense of values.
- Pursues interest with tremendous energy and drive.
- Uses his spare time productively.

Special Characteristics

- Frequently impatient with drill and details that he thinks are not important.
- May be reading mathematics books years ahead of his age.
- Recognizes patterns readily and enjoys speculating on generalizations.
- Prefers to think on higher levels of abstraction.
- Classifies particular cases as special cases of more general situations with relative ease.
- Follows a long chain of reasoning, frequently anticipating and contributing.
- Frequently asks profound questions.

7.4.2 Enrichment Programme for the Gifted

The gifted children have tremendous energy with a lot of determination to realise the goals. If not directed properly, this reservoir of energy may go waste and sometimes may create serious problems for the individual and society. The idea of giving special attention to the gifted children by arranging separate classes or sections is not practicable as the number of such students in a particular school, class and subject may be very small. Moreover this amounts to special treatment to a few at the cost of many and looks undemocratic. The other alternative is to collect all of them at one place in a

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district or region. Among average companions he will remain at the top without much effort and will not face any competition. When collected at one place in an ideal school brought into the company and competition with classmates of their own level and are placed under the charge of selected and really competent teachers, they will be able to do their best. Such a school will be residential where all round development of these students may be ensured. These schools can also be equipped adequately to cater to their various and special educational needs. A truly academic inspiring scholarly and dedicated atmosphere can alsobe created.

For providing additional learning opportunities, under the enrichment programmes, the following two channels are suggested.

- *Differentiated curriculum for the gifted*
  - A curriculum which is more challenging can be devised for the gifted students.
  - Such curriculum should contain more advanced topics and challenging tasks.
  - The differentiated curriculum should provide opportunities for the students to explore, investigate, critically analyse, reason out and discover mathematical ideas and facts independently.
  - A parallel track curriculum (*Example*: 'A' level and 'O' level) will provide the necessary flexibility.
- *Enrichment within the existing curriculum.*
  - Under the second scheme, attempts may be made to provide enrichment programmes and opportunities within the existing curriculum.

The following steps may do greater justice to them.

- A differential curriculum providing an enriched syllabus to motivate the mathematically gifted students.
- Differential assignments such as multiple level and contract type assignment which are challenging and stimulating.
- Opportunities for independent and original work demanding extensive reading, use of cyber resources and so on.
- More scientific and sophisticated evaluation tools to assess the performance of the students, so that the evaluation is reliable and valid.
- Adopting teaching methods such as Project method, Analytic method, Heuristic method, Discovery method and Problem Solving method, so that the learning process involves participation and independent thinking.

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- More time can be allotted for independent study through projects, seminar and assignments.
- As the gifted students do not require drill and practice, routine problems can be avoided. Instead challenging and thought provoking problems should be presented to them.
- Mathematically gifted students should be trained to take up mathematically challenging tasks such as organising mathematics exhibition, preparing models, presenting papers, solving puzzles, writing articles and so on.
- They can be encouraged to participate in panel discussion on topics in mathematics and quiz competition in mathematics.
- In solving mathematical problems, the gifted students can be encouraged to try and use alternate methods of attack and any such attempt should be appreciated and recognised by the teachers.
- The gifted students can be involved in the supervised study and tutorial classes for the slow learners in mathematics.
- The gifted students can be made responsible for organising the school mathematics club and its activities.

The gifted students can be encouraged to participate in National Talent Test such as National Talent Search Scheme conducted by NCERT and Mathematical Olympiads such as International Mathematical Olympiad (IMO), the Mathematical Olympiad Programme organised by National Board of Higher Mathematics (NBHM) of the Department of Atomic Energy etc. Such participation will provide ample opportunities for the students to quench their thirst for knowledge and face real mathematical challenges.

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National Talent Search Examination

Frequently Asked Questions

1.	**GENERAL**	
i.	What is this scheme about?	This scheme is aimed at identification of talented students and awarding them scholarship to pursue their further studies and nurture them.
ii.	How old is this scheme?	The scheme is almost as old as NCERT; In 1963 the scheme was introduced as National Science Talent Search Scheme.
Iii	How long successful candidates will be getting scholarship?	The awardees studying in Sciences, Social Sciences, Humanities, Languages Commerce, vocational studies and fine arts are eligible to receive scholarship up to Ph.D. level whereas the awardees studying professional courses in medicines, engineering, technology, management and law, are eligible to receive the scholarship up to second degree level.
iv.	How talented students are identified?	Talented students are identified through two- tier examination process.
v	What is this two -tier examination scheme?	This is a scheme in which exam is conducted in two stages.
vi.	Who conduct first level examination?	NTS Stage-one is conducted by states and union territories. The purpose of this exam is to screen the number of candidates for Second level exam to be conducted by NCERT.
vii	Who conducts this second level examination?	National Council of Educational Research and Training (NCERT) conducts this examination.
viii	What is the criterion for selection to stage-II examination?	There is quota earmarked for all the states/ UTs. The state quota is computed on the basis of the student's enrolment at secondary level with a minimum of 10 for union territory and 25 for a state.
ix	When can one appear for NTS Stage-I Examination?	Currently NTS Scheme is open to the students who are presently studying in **Class X.**
x	Does this scheme available for other classes also?	No.
xi	How many scholarships are awarded every year?	Every year about 1000 scholarships are awarded.
xii	Is there any reservation for scholarships?	Yes, there is a provision of reservation for SC and ST candidates based on national norms of 15 percent and 7.5 percent

		respectively only those candidates who obtain minimum qualifying marks under these categories are eligible for scholarship.
Xiii	Does the scheme have provision of reservation for physically challenged students?	Yes, there is a provision of 3 percent reservation for physically challenged students.
2	**SELECTION PROCEDURE**	
i.	Can my son/daughter who is studying in India write the NTSE stage 2 Exam directly?	No, In order to appear for stage II exam the candidate name must be recommended by the State/UT to which he/she belongs.
Ii	Who can appear in this exam?	All students studying in Class X in any type of recognized school including, Kendriya Vidyalayas, Sainik Schools, Military Schools will be eligible to appear at the State Level Examination from the State in which the school is located.
Iii	Are there any eligibility conditions for appearing in the screening examination like qualifying percentage of marks in the previous year annual examination etc.?	Eligibility criteria for stage I varies from state to state. The State/UT may impose any eligibility condition for appearing in stage I examination.
3	**STATE LEVEL EXAMINATION (STAGE 1 EXAM)**	
i.	When will the notification be given?	For stage I examination, advertisement is released in the month of July- August.
ii.	Where can I get the application for Stage 1?	You need to contact Liaison Officer of your State/UT. Addresses of Liaison Officers are placed on the NCERT i.e. (www.ncert.nic.in)
iii	Do all the states/UTs conduct stage I exam every year?	Yes, each state/UT conducts its own stage I examination.
Iv	When is stage I examination held?	The state level screening examination is normally conducted in all the states/UTs on second Sunday of November except in Nagaland, Andaman and Nicobar Is. Meghalaya and Mizoram, where it is conducted on second Saturday of November every year until and unless some special circumstances occur.
v.	When to appear for Stage 1 NTSE?	One can appear for Stage I examination in the month of November for which you need to apply in the month of July-September (as per advertisement of each State/UT.)
vi	Should we apply through school or directly?	You need to submit your application after getting it duly signed by the

		Principal of the school before the due date as advertised/circulated by your state/UT.
vii.	How can I apply for stage I examination?	If you are a student of class X of a recognized school then you need to look for advertisement or circular in the school by the respective State/UT Government for the above said examination and act as per the requirement given in the state advertisement/circular.
viii.	If I have some other queries related to stage I examination, then whom should I approach?	In case of any other information /query about the details of the state level examination you may approach the Liaison Officers of your state/UT. **NCERT does not entertain stage I applications**
ix.	Can we send our application directly to NCERT?	No, you are advised to submit your application to your Liaison Officer of State/UT.
x	What will be the medium of examination at stage I?	The medium of the examination shall be as announced by the State/UT.
xi.	Is there any fee for appearing in stage I examination?	You need to check from state advertisement/circular. Each State/UT has its own rules with respect to charging of fee.
xii	What is the scheme of examination at Stage I?	Stage I Examination normally has three parts: Part I Mental Ability Test (MAT), Part II Language Test, Part III Scholastic Aptitude Test (SAT)
xiii	When state level examination results are are announced?	The results of State Level Talent Search Examination is declared generally in the months of January/February by the States/UT.
xiv	Are marks scored in stage I examination added to the stage II exam conducted by NCERT?	No.
xv	In case of any query/complaint/clarification with regard to State Level NTS Exam whom shall I contact?	You may correspond only with the state examination agencies (Liaison Officer).
4.	**NATIONAL LEVEL EXAMINATION (STAGE II EXAM)**	
i.	Is there any fee for appearing in the National Level Examination?	No, such fees is charged for appearing in NTS Stage II examination.
ii.	Who can appear for NTSE stage II examination?	Only the students selected by the states/UTs on the basis of their screening examination shall be eligible to appear in the NTSE stage-II to be conducted by the NCERT.
iii.	When stage II exam is	NTSE Stage II examination is conducted

	conducted?	on the second Sunday of May each year (unless otherwise notified).
iv.	I have cleared stage I examination, Do I have to fill and submit another form for stage II examination?	No, you need not fill and send any form for stage II examination.
v.	Whom should I contact for my Roll Number for stage II examination?	There is no need to contact anyone The NCERT will convey the Roll Number, the venue and the time for the national level examination to all such candidates directly through speed post.
vi.	My son/daughter is studying abroad; can he/she appear for NTS Examination?	Any student of Indian nationality studying abroad at the class X level can appear, provided she/he has secured 60 % marks in the previous examination, will have to appear in the NTS examination center in India at her/his own cost. Such candidate may send the filled in application request through head of the institution where he/she is studying along with an attested copy of class IX. Such requests can be made to Head Educational Survey Division, NCERT, New Delhi-16 within stipulated time. Such candidates are exempted from stage I examination and are permitted to directly appear for stage II examination. The NCERT shall allot roll numbers to such eligible candidates. If such a candidate is selected, the scholarship shall be paid for pursuing studies in India only.
vii	What will be the syllabus for NTS Stage II examination?	There is no prescribed syllabus for the NTS Stage II examination; However, the standard of items shall be conforming to the secondary level.
viii	How many papers will be there in stage II? How many questions will be asked in these papers? What will be the time limit for these papers?	For stage II NTS examination, there are three papers, Mental Ability Test (MAT) of 50 questions , (45 minutes), Language Test of 50 questions (45 minutes) and Scholastic Aptitude Test of 100 questions (90 minutes)
ix	Is there a negative marking?	Yes, there is negative marking. For each wrong answer 1/3 marks will be deducted and no marks will be deducted for unattempted questions.
x	Language Test is conducted in how many languages?	In Stage II NTSE Language Test is conducted in Hindi and English. Candidates need to choose either Hindi or English.
xi	Is language Test a qualifying	Yes, Language Test is qualifying in

	Test?	nature. That means marks scored in language test will not be counted for final merit.
xii	Do candidates get separate answer sheet to mark answers?	Yes, in all the three papers OMR answer sheets are provided.
xiii	I am from Assam, will I be getting question booklet for SAT and MAT in Asamiya?	Yes, you will be provided with question booklet in Asamiya language. The tests are also available in Asamiya, Bangla, English, Gujarati, Hindi, Kannada, Marathi, Malyalam, Odia, Punjabi, Tamil, Telugu and Urdu.
xiv	Whom should we contact for seeking information about second stage roll number and venue of exam?	There is no need to contact anybody. NCERT will directly send you roll numbers for Stage II NTS Exam by speed post. The letter will also indicate venue of examination along with other details.
xv	I have shifted to new location after my stage I exam, how will I get my roll number?	In case there is any change of address prior to the national level exam, it shall be obligatory on the part of the candidate to communicate the same to the state examining authority, which in turn shall inform NCERT.
xvi	Do we have center in all the state capitals in the country?	There are 36 centers across the country. Normally the candidates belonging to a particular state shall be allotted the center in the same state for the National Level Examination.
xvii	After my stage I examination, my family had shifted to another state, in that case I want to opt for center in my present state, for that whom should I request?	Only in exceptional cases under special circumstances like the transfer of the parents of the candidates, the center for examination may be changed on a written request received by the NCERT within 15 days of the issue of the Admission Card and 10 days prior to examination. It is not obligatory for NCERT to provide change of center in any case. However in such case Test Booklet in English Medium will be provided to candidates.
xviii	Is there any interview in the examination schedule?	There is no interview. Selection is based on the performance of written test in MAT and SAT and obtaining minimum marks in all the papers Language Test.
Xix	When results are declared?	Normally results are declared in the month of August.
Xx	How will I come to know about results?	Results will be uploaded on NCERT website. Only the selected candidates shall be informed by letters sent through speed post.
xxi	Is there any provision of rechecking or retotalling?	NTSE stage II examination is computer processed. NCERT takes extreme care in

		result processing. There is no provision for rechecking and retotalling. Al the OMR answer sheets are placed on web portal to maintain the transparency, candidates can see only their OMR answer sheets.

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MATHEMATICAL OLYMPIAD

History of mathematical Olympiad:

A mathematical Olympiad problem solving competition open to all 'athletes' the aim of the competition is to test innate problem solving skills. It is created in 1977 by Dr.George lenchner, an internationally knows mathematics educator, mathematics O

The main international mathematical Olympiad is the international mathematical Olympiad (IMO), a competition held annually since 1959.

Objectives of Mathematical Olympiad:

1. To stimulate Enthusiasm and love and interest for mathematics.
2. To introduce important mathematical concepts.
3. To teach major strategies for problem solving.
4. To develop mathematical flexibility in solving problem.
5. To strengthen mathematical instruction
6. To foster mathematical creativity and ingenuity.
7. To provide for the satisfaction, Joy and Thrill of meeting challenges.

Organization of the mathematics Olympiad programme in India:

The mathematical Olympiad programme in India leading in participation in the IMO is currently being organized by the "Homi Bhabha" center for science education (HBCSE) on behalf of the national board of higher mathematics (NBHM) and founded by the department of Atomic Energy (DAE).

The correct office bearers in charge of the programmes are:

- National Co-ordinate-Delhi
- HBCSE Director – Mumbai
- Member Secretary, NBHM – Mumbai
- Scientists' In-charge-Bangalore.

Procedure for Participation:

There is a three-step procedure in order to represent India of IMO.

1. Regional Mathematical Olympiad (RMO):

In India, CBSE & Navodaya Vidyalaya samiti are designated and also 21 different regions in country are designated. All school students' classes XI & XII are eligible to appear in RMO Exam held b/w September and the December.

2. Indian National Mathematical Olympiad (INMO):

INMO is held on 1st Sunday of Feb. each year at center of each region. INMO is open only to those selected through RMO from their respective regions or through RMO conducted by CBSE or NVS.

3. International Mathematical Olympiad Training Camp (IMOTC)

The awardees of INMO are invited to a month – long training camp held in May, June. Each year at HBCSE Mumbai. Training facility from all over the country imparts problem solving skills along necessary theoretical background to the awardees.

Syllabus for Mathematical Olympiads:

There is no prescribed syllabus for mathematical Olympiads. The topics are taken from pre-college mathematics. The areas covered are Arithmetic of Integers, Geometry, Quadratic equations and expressions, Trigonometry, co-ordinate Geometry etc., the major areas from which problems are chosen are number theory, Geometry, Algebra & combinatory etc. The difficulty level increases from RMO to INMO to IMO.

Importance of Mathematics Olympiad

1. It Provide opportunity for gifted children.
2. It develops competitive spirit.
3. It identifies gifted children.
4. Encourage and award gifted children.
5. Expose children to regional, national and international levels.
6. To help children to know their capabilities.
7. To nurture their talents.
8. To enhance their critical and problem-solving skills.

Example problems for Mathematical Olympiad:

1. Let a, b, c be +Ve real numbers, such that abc=1 prove that (a-1+1/b) (b-1+1/c) (c-1+1/a) < 1
2. Deter mine all functions f; R-R such that f (x-f(y)) = f(f(y))+x+(y)+f(x)-1 for all real numbers xy
3. Find all primes pand q such that, p2+7pq+q2 is square of an integer.
4. Find all real values of (a) for which the equation $x4-2ax^2+x+a^2-a=0$ has all its roots real.
5. Let ABC be a in which AB = BC and CAB= 90^0 suppose M and N are points on the hypothectic On use BC such that BM+CN=MN, Prove that MAN=45^0
6. Special Programmes in Teaching Mathematics

1. Defects in the present day Teaching Mathematics in schools and their possible Remedies.

It should be frankly admitted that the present-day teaching of Mathematics is far from being satisfactory.

Everybody has a complaint against the teaching of mathematics. It is dull, boring, difficult and useless from the point of view of the learner. “It is too remote from life to the student”. The teachers complain of excessive work load and lack of facilities in the form of aids and equipment.

Defects and possible Remedies:

1. Teacher’s Qualification:

It is a common defect in our educational set-up that most of the subject teachers are not adequately qualified in the subject concerned, without proper qualification and proper training. They fail to do justice to the subject. An adequate, high qualification of the teacher develops self-confidence in him and serves as a source of inspiration to his students. The teacher must be

nature in his subject. He must possess real knowledge of and insight on to the processes of mathematics and their effective teaching.

2. Teachers Borden:

Teacher cannot adopt and prepare for, effective methods, as he has no spare time. His burden does not allow him time to remove individual difficulties. It should be lightened to enable him to show his originality and initiative.

3. Teachers Salary:

Mathematics or other teacher economic position is not good. He remains worried, and a worried teacher cannot give his best to the learners. He is a frustrated, discontented and half-hearted worker. In these hard days, he must be suitably paid.

4. Teacher's Attitude:

Teacher does not have genuine love for his mathematics subjects and profession. He lacks faith in the utility of the mathematics subject, and therefore, cannot create interest among the students. A teacher's love for his Job and subject should also be ascertained before giving him his duty.

5. Lack of purpose:

The students do not recognize the purposes behind the study of the topics of mathematics. The particular and general aims of every topic should be emphasized effectively. If the work lacks purpose, it is the teacher's duty to make it purposeful. The purpose should be attractive to stimulate the students to work hard. This misconception should be up rooted from the minds of the parents and pupils that most of the mathematics taught in the schools is not purposeful.

6. Lack of Equipment:

There is a serious lack of mathematical apparatus in the schools. Without equipment, the subject becomes abstracts. The establishment of a mathematical laboratory will remove this defect.

7. Method of Teaching:

The powers of thinking, understanding and retention are not thus developed in the students. If the pupils do not show any interest in the subject (maths) if can be created not by blind memorizing, but by shifting the methods. The authorities run after showy results which are obtainable only through cramming. They have no appreciation for good mathematical teaching. The remedy necessitates a fundamental change in value and methods.

8. Large classes:

It is a general defect. No individual attention cannot be paid. It becomes difficult for the teacher to establish close contacts with the students. Teacher cannot easily Judge the capacities of the individuals. This defect can be removed only by limiting the number of students in each class up to a maximum of Thirty-five.

9. Text – Books:

The traditional style of the syllabus also affects text-books adversely. The illustrations and problems give in the text-books are divorced from actual life. The mathematics text-book material is made available in a readymade form which goes against thinking, discovery and originality. Text books should give possibilities of correlation, application in practical life, use of

aids, activities, projects etc., concerning every topic in mathematics. The arrangement of the subject matter should both and laboratories.

10. Libraries and Laboratories.

11. **Examinations.**

12. Syllabus

13. Mathematical Language.

14. Rigor in study etc.

Even after removing the above mentioned defects completely there will be a scope for improvement

DIAGNOSTIC AND REMEDIAL TEACHING

Generally mathematics teaching is organized to facilitate average students. The individual variation of the students is taken in to consideration. Thus, the extreme case-bright students and poor students are ignored in the normal teaching of mathematic teaching. This chapter provides the awareness about diagnostic test in mathematic teaching and planning for remedial mathematics teaching.

Diagnostic Tests

Meaning of Diagnostic Test

It is a form of achievement test. A diagnostic test is one kind of educational test. It is designed to reveal specific weakness or failures to learn in some subject of study such as reading or arithmetic. The difficulties of students can not be easily identified from this test. The aim of diagnosis is to analyse the difficulties and weakness of a student in a particular phase of work. Through diagnostic devices, efforts are made get reliable informations and reasons concerning the weakness of the student in order to overcome them by concentrated action and for detecting needs for remedial teaching.

The word 'diagnosis' is extensively used in medical science. The patient is prescribed medicine on the basis of the diagnosis. If the diagnosis is proper, the medicine prescribed will cure the patient soon. In the same way, in the field of education, if the proper causes of not learning a subject matter correctly by the students are found out, remedy too can be done properly.

In a diagnostic test the main interest is the performance on individual item or on small groups of highly similar items. In this test score or marks is not assigned for the correct answer but wrong answer provide the basis for the causes of his failure. Diagnostic tests are qualitative not the quantitative. A diagnostic test does not yield the total scores of an individual in a subject which he has studied and taken the test.

Definitions :

According to **Good,** "Diagnosis means determination of the nature of learning difficulties and deficiencies."

According to **Ross,** "Prevention is the highest level of diagnosis."

Again **Ross say that,** "The diagnosis value of a test depends more upon the teacher than upon the test used."

According to **Yoakum and Simpson,** "Diagnosis is the art or act of recognizing a difficulty by its symptoms. It is an explanation of the difficulty based on an examination of the facts."

According to **Good & Brophy,** "Diagnostic teaching refers to the process of observing student responses carefully to diagnose the specific nature of difficulties in learning."

Hence, the diagnostic test is the test which is constructed in order to find out the short comings and hindrances coming in the way of learning a unit of a subject matter and on the basis of these findings provides remedial suggestions to do away these shortcomings and hindrances.

Purposes of Diagnostic Test

The main purposes of diagnostic test are :

1. To give suggestions in improving the teaching process of teachers.
2. To give suggestions for effective evaluation process.
3. To suggest to make the curriculum more useful.
4. To arrange the remedial teaching.
5. Make an effective teaching - learning situation.
6. To help in selecting the different type of questions for the construction of achievement test.

Functions of Diagnostic Test

The functions of diagnostic tests in schools can be as below :

1. To find out the bases for improvement in teaching process.
2. To suggest to make the text books more useful.
3. To identify the backward students in learning a specific subject, and to suggest remedies for them.
4. To change evaluation methods to make the evaluation process more effective.
5. To provide educational and vocational guidance on the basis of difficulties and hindrances of students.
6. To find out the hindring factors in learning process and affect necessary changes accordingly.
7. To find out the difficulties and hindrances of students in learning and to suggest remedies.

Characteristics of Diagnostic Test

The main characteristics of diagnostic test are :

1. These tests are both standardised and non-standardised.

2. There is no limitation of diagnostic test.
3. It is a main part of curriculum in diagnostic test.
4. These tests are constructed according to specific objectives.
5. These tests are analytical.
6. These tests specify the mental condition of the students.
7. The progress of students is tested objectively by these tests.
8. It is an effective tool for teachers that helps in planning and organizing remedial teaching.
9. It finds out weakness of a child in learning of a content.
10. In diagnostic test no scores is made for correct answers, only wrong responses are taken into view in the sequence of contents.

Construction of a Diagnostic Test

For a good or proper construction of diagnostic test there is need of proper planning. For this purpose we should keep in mind the following things :

1. Formulation of objectives and outline of the content or topic.
2. Content analysis is divided into sub-topic and its elements:
   (i) Sequence of sub-topics and elements within the sub-topic.
   (ii) Sequence of learning points.
3. Identify difficulty in order of sub-topic.
4. Deciding the types of the items.
5. Preparing items and tryout.
6. Item analysis of test items and modification of items.
7. Analysis of logical sequence of content by preparing scalogram.
8. Preparing the final draft of the test.
9. Preparing manual of the test.
10. Remedial devices or measures.

Process of Diagnostic Test

It is completed in five steps :

1. Selection of diagnostic students.
2. Identifying difficult points
3. Analysis of difficult points
4. Remedial process
5. Preventive measures

1. Selection of Diagnostic Students–In this step, those students are selected who are weak in one or more subjects, who have problem in adjusting with the school. Such students are selected by conducting, achievement tests, intelligence tests, interview and observation.

2. Identifying Difficult Points–In this step, the difficulties of the students are found out by interview, observation, intelligence tests or achievement tests. The teachers experience of informal tests conducted by them, observation and interview prove more effective in it.

3. Analysis of Difficult Points–In this step, the teacher finds out the causes of difficulties of the students. The teacher attempts to find out the causes of these difficulties by his experiences, conversation with the students, fellow teachers and guardians. The origin of these causes can be physical defect, mental instability, bad habits and mental retardation etc.

4. Remedial Process–After having diagnosed the weakness of the students, remedial measures are adopted. Suitable plan is made to eradicate them, which describes the causes of difficulties and the remedial measures. Besides, it also mentions whether the difficulty has to be remedied collectively.

5. Preventive Measures–The causes of difficulties are analysed, planned effort is made so that they commit less errors in the future. The preventive measures may include one or more measures out of improvement in school atmosphere, improvement in home atmosphere, amendment in the curriculum, improvement in the examination system, etc., which may reduce the errors in the future.

Precautions Regarding Construction of Diagnostic Test

Teacher should observe following precaution in the preparation of diagnostic test :

1. Teacher should construct the diagnostic test ownself.
2. Should give more emphasis on the typical or difficult words used in teaching process.
3. These test should construct the difficulty level of the content.
4. Students do not repeat the demerit of previous.

Remedial Teaching

Remedial measures should be adopted to remove the weakness and difficulties experienced by the student in a specific field of mathematics. It is selected the weaknesses of student through diagnosis. So, the diagnostic test is only a means of finding out the weakness of student and the reasons behind them. After eliminating the factors, remedial teaching should be done. The mathematics teacher may also prepare corrective material for this purpose. Thus, by remedial teaching the success can be achieved in removing the weaknesses of the students.

Definitions :

According to **Yoakum and Simpson Boserve,** "Remedial teaching

is actually old, since good teachers from times immemorial have always tried to correct the errors in children's reading and set them on the right track. The new remedial teaching has received as stimulus form the testing movement, however and has been given a name borrowed from the medical profession. It has for its purpose the development of effective techniques for the correction of errors in all types of learning. As yet, it has been more effectively used in the skill subjects than in the mathematics. This is probably because the errors in learning may be more easily detected in the former than in the later."

According to **Yoakum and Simpson,** "Remedial teaching logically follows diagnostic teaching."

According to **Mc Millan Co (1956) (Diagnostic & Remedial Teaching, New York),** "Remedial teaching is concerned with two types of deficiencies, the presence of bad habits and the absence of good habits."

According to **Blair, Jones & Simpson,** "Remedial teaching is essence, is just good teaching that takes the learner where is and through well motivated activities leads him to increased competence in his areas of weakness.

Functions of Remedial Teaching

Following functions of remedial teaching have been indicated :

1. To solve the learning difficulties of the poor students.
2. To assist the poor students to pace with his classroom teaching.
3. To avoid the wastage in education process.
4. To raise the standard of mathematics teaching.
5. To evolve the effective strategy of mathematic teaching.

Aims of Remedial Teaching

The main aim of remedial teaching are as follows :

1. It is used to remove weakness of the child.
2. Give personal and group discussion for knowing the errors of the students.
3. To avoid the learning-related defects and drawback.
4. To give proper direction.
5. To change the undesirable interest, ideals and point of view to desirable interest, ideals.

Yoakum and Simpson defined the aim of remedial teaching in this form :

"Remedial teaching has for its purpose the development of effective technique for the correction of errors in all types of learning."

—Yoakum & Simpson

Principles of Remedial Teaching

1. To improve the relation between the teacher and students.
2. In this the result of progress can be achieved fastly.
3. In the process of remedial teaching students are more active.
4. For this purpose teacher should be trained or expert in diagnostic test.
5. In this teacher should be broad point of view in his experiences.

Procedure for Remedial Teaching

Procedure for remedial teaching are :

1. Identify the poor students–First step is to identify poor students in mathematics subject by using achievement test, school marks, personal observation of teacher and interview technique.

2. Diagnostic Test–In second step, a diagnostic test in mathematics related to specific area, pronunciation, spellings, reading, grammar etc. is administered to locate the learning difficulties and its causes.

3. Remedial Teaching–In this step remedial instructions are to be prepared for remedial teaching.

4. Strategy–An appropriate strategy is used for remedial teaching. After this, a test is administered to ascertain, how for learning difficulties could be removed. It may suggest about reteaching or further remedy.

Remedial Teaching in Mathematics

In mathematics teaching gifted children and dull children give separate remedial teaching.

A. Remedial Work for Dull Students

Dull student concentrate his mind very less in any work. So he needs individual instruction. So, following remedial teaching should kept in mind for giving education to dull students :

1. He should sit infront in the class.
2. Pay more attention to solve the mathematical problems. For this purpose they should be taught the concept, theories etc.
3. To understand any concept, theories etc. teacher should use chart, model, diagram etc.
4. Give more emphasis on law of exercise.
5. Teacher should check the home work or any written work of dull student precautions and tell their mistakes.
6. Teacher should select the proper examples related to topic in a class room teaching.
7. Give more emphasis on practical work.

8. Teacher should use effective question-answer method in classroom to motivate the dull students.

B. Remedial Teaching for Gifted Children

Gifted children are more active. They are different to normal children, for this purpose they need the special education and training. They adjust in class with difficulty.

According to **Kolesnik,** "The term gifted has been applied to every child who, in his age group is superior in some ability which may make him outstanding contributor to the welfare and quality of living in our society."

Following points should be kept in mind for teaching of gifted children:

1. Separate schools should be made for these children with proper facilities which fulfil the development opportunities of these children. There can be provision of special schools for talented children.
2. Arrange special examinations for evaluation.
3. Use more effective method as question answer methods, group discussion method, project method etc. in teaching.
4. These children are interested in basic studies. To develop this interest libraries should be maintained.
5. Encourage these students to help the dull students.
6. Gifted children should be assigned special home work so that they may utilize their talents in a proper way.
7. These children are more thirsty for knowledge in comparison to the normal children. Since teacher should provide different facilities to the talented child for this purpose they provide library facilities.
8. Teacher should prepare the improvised apparatus related to mathematics.

Precautions Conducting for Remedial Measures

The following factors should be kept in view while conducting remedial measures :

1. The weak student should be asked to sit on the front seat in the class.
2. The development of the subject matter should be done with the solved examples and illustration.
3. The students should be given individual counseling even after the class, to help them in learning.
4. The fundamental concepts of mathematics and other subjects, such as factors, percentage, unit, square root, etc. should be taught carefully.

5. The exercises on each sub-topic should be such which the students can think about themselves.
6. The correction in the written work of the students should be done in their presence.
7. The student should be provided sufficient opportunity for thinking and reasoning in the class.
8. The attention of the students should be drawn to those concepts, principles and activities related to the subject matter in which they commit errors.
9. The concepts should be provided sufficient opportunity for thinking and reasoning in the class.

Organisation of Remedial Teaching

A teacher can organise remedial teaching in a school in a following ways :

1. Class Teaching
2. Group Tutorial Teaching
3. Individual Tutorial Teaching
4. Supervised Tutorial Teaching
5. Auto Instructional Teaching
6. Informal Teaching

1. Class-Teaching–In this, there is no change in the present structure and form in formal teaching organisation. In this teacher do work and teaching in a traditional way. In the class teaching, teacher shoud know that the students has difficulty of which branch, topic, content and process etc. Teachers should keep in mind that their new teaching is based on the previous knowledge of the students.

2. Group Tutorial Teaching–In this teaching students are divided in a group of a special characteristics. This division is based on their difficulty levels, similarity of problems and weakness. After this each group are taught by the teacher according to their difficulties. For each tutorial group an incharge is appointed, who select the intelligent students in these groups.

These tutorials are useful for an average student. These can satisfy the needs of less exceptional students. The teacher should have the background of mathematical and social psychology and group dynamic so that he can deal with group tutorial effectively in solving their problems.

3. Individual Tutorial Teaching–In this teacher pay attention individually to the students and solve the problems, difficulties etc. in individual.

4. Supervised Tutorial Teaching–It is also called supervised study method. In this method the teachers remove the defects of traditional methods as–lecture, exploration etc. In this teacher works as a supervisor. In this type of tutorial students and teacher's meeting are arranged regularly. A student reads an essay and defends it with arguments. This strategy can provide an opportunity to deepen his understanding of subject for able student. It stresses on the mastery over the basic skills of scholarship. Teacher can be use in the two form :

(a) Group form

(b) Individual form

In this teacher can supervise the students in a group or an individual

5. Auto Instructional Teaching–In this students remove their demerits and difficulties by auto instructional. The main characteristic of this are:

(i) Student is not to receive any supervision or guidance.

(ii) Give the programmed learning package to solve the difficulties related to subjects. This is a auto instructional material or aid.

(iii) This auto instructional aid can accept in a form of computer software programme.

In this way student solves his difficulties by self.

(b) Informal Teaching–In this there is no fix place, time, curriculum. The subject related knowledge are added into formal education. In the informal teaching include following programes:

- Educational excursions
- Organisation of Mathematic club
- Construction of science model
- Mathematics Museum
- The equal role of social and cultural programmes.
- Collection of different things related to mathematics.
- Participation in group decision and debates.

Construction of Diagnostic Test in Algebra

Name of Student : Abhishek Singh — Class - 7

Name of College : Government Inter college, Roorkee — Section - B

Date : 17 Nov. 1977

Instruction—

1. Write the value of 'a' based on simple equations.
2. Write the answer in appropriate place.
3. If appropriate place is short then you take another page to rough work.

Questions	Answers	Place of Calculation
1. a +6 = 28 a =	(1)	
2. 8a + 10 = 26 a =	(2)	
3. 3a + 23/5 = 74/9 a =	(3)	
4. 0.05 a + 0.6 = 6.8 a =	(4)	
5. 0.6 a + 8 = 5.3 a =	(5)	
6. 6a – 12 = 6 a =	(6)	
7. 8a – 0.14a = 43 a =	(7)	
8. 0.023a – 7 = 9a – 3 a =	(8)	
9. 12a + 4 = 4 – 4b a =	(9)	
10. 4a + 2 = 0.8a + 1 a =	(10)	
11. a/4 – 3a/5 = 6 a =	(11)	
12. 4a + 3/8a = 8 a =	(12)	
13. a + 2a/3 = 15 a =	(13)	
14. a + 10 = 22 a =	(14)	
15. 9a = 27 a =	(15)	
16. 4a = 1/9 a =	(16)	
17. 4a – 8 = a + 8 a =	(17)	
18. 6a + 7a = 33 a =	(18)	
19. a/2 + 1 + 1/2 =5/2 a =	(19)	
20. 4 (a + 4) = 16 a =	(20)	

All these questions are written on the basis of 'simple to complex' form. The main aim of this exam is to solve the difficulty level of the class 'Y' students that they solve how many questions and how many questions have they difficulty?.

Diagnostic Chart

	1	2	3	4	5	6	7	8	9
Name of Student	Subtration in Equation (2)	Multiplication (1)	Equation and simple diffrection (4)	Equation and decimal Bhire (3)	Equation and Simple Total (4)	Pakshantar (6)	Score	Absence in Class	Special Description
	(2)	(1)	(4)	(3)	(4)	(6)			
(1) Deepak Kumar	–	–	–	–	–	–	20	2	I
(2) Pratibha Rani	–	–	–	2	1	–	17	5	III
(3) Seema Sharma	–	–	–	–	–	1	19	3	II
(4) Imran	1	–	4	2	–	1	12	7	V
(5) Yasoda Kumari	1	–	–	1	2	6	10	12	VI
(6) Vinod Kumar	–	2	–	–	1	–	17	4	III
(7) Rashi Arora	3	–	–	–	–	1	16	6	IV
(8) Total Error of total Students	9	–	–	12	14	17			
(9) No. of correct question done by the 40 class students	46	32	58	43	86	74			
(10) Total of Questions	60	35	60	70	100	90			
(11) Percentage of correct questions	76	91	96	62	86	82			

Interpretation—Above chart analyses the weakness and difficulties of the seven students. In present chart Yasoda Kumari is a weak student in a class. The get 10 marks out of 20 marks, and they have more absence in the class.

Deepak Kumar is a good student of his class he got 20 mark out of 20 which slow that they understand the subject. Seema Sharma also a good student, they correct 19 questions out of 20.

Formative Evaluation

When the students are said to be in well state regarding knowledge the process of teaching applied to learn any thing is called formative. The methods of judging how much the reactions of teaching are made to students is evaluation. The purpose of formative evaluation is to monitor the instructional process to know that learning process is taking place or not. It is designed only to enhance the teaching-learning process and not designed to make final judgements.

Hence, in formative evaluation teaching process is made and learning is judged. The process works upon in a long term, so it is called continuous evaluation in which unit tests, class tests and assignments are the essential components.

Formative evaluation provides feedback to children, on the other it aparts feedback to teachers also, so as to, modify instructions to prescribe remedial work. Therefore it can easily be said that formative evaluation brings improvement in child's learning and also removes the drawbacks that are committed during teaching and learning process, hence teaching-learning process is better enhanced through this evaluation.

Formative evaluation can give the relevant information during class room teaching-learning process.

According to **Tanner, (1972),** "Formative evaluation refers to the use of tests and other evaluative procedures while the course and instructional programme is in progress."

Need and Importance of Formative Evaluation

1. It provides informations to teachers for modifying instruction and teaching.
2. Helpful in diagnosing child's strength and weakness.
3. It is helpful in monitoring pupil's learning progress.
4. It facilitates retention and transfer of learning.
5. It can also be used as a self-evaluation device by the children.
6. It helps to prescribe group and individual remedial programs.
7. It provides immediate feedback to the children.
8. It also reinforce the learning of high achievers.

Summative Evaluation

Summative test is given to the student after he has passed successfully all the formative tests given separately in the last of each unit of he content. Overall results of teaching-learning process can be obtained through this short of evaluation. Simple class test, unit tests, quizzes and learning tests are the parts of formative evaluation, then term test-annual tests and external examinations conducted by school or public agency are the essential parts of summative evaluation.

Hence, summative evaluation is either external type or Internal type evaluation. In other words, it can also be said that formative evaluation serves refinement purpose during teaching-learning process, but summative evaluation may be said the evaluation of refined product.

Summative evaluation becomes part and parcel with future point of view. It is identification document of failure and success of the class indicated. So, summative evaluation is inevitable and has to remain an integral part of the whole evaluation process.

Difference between Formative and Summative Evaluation

Formative Evaluation	Summative Evaluation
1. The process of this evaluation is to diagnose the strength and weakness of the pupils.	1. The process is for classification and promotion of student.
2. Its focus is an improvement of pupil's achievement.	2. Its focus is on measurement of pupil's achievement.
3. It refers to continuous evaluation by means of unit tests, assignments etc.	3. It refers to term tests, annual tests and external examinations.
4. It is an integral part of teaching-learning process.	4. It is treated as an end of the course activity.
5. It is done during the course of instruction.	5. It is done at the end of academic session/years.
6. It gives relevant informations which helps to improve classroom instruction.	6. It provides the overall results of the teaching-learning process.

Co-operative and collaborative strategies:

Collaborative Learning

This is a form of learning which shares the learning responsibility among the members of a group which works towards a common goal. The common goal can be achieved only when all the members perform their duties satisfactorily. The outcome of learning can be shared by all the members of the group. The teacher who facilitates learning need to clarify the responsibilities of each member. This can be done only through discussions with the members of the group. The situation where one person acts on behalf of the group can be avoided in this type of learning. Collaborative learning method can be followed only by a teacher who is prepared to share knowledge and authority with the students.

Collaborative activities are most often based on four principles:

- The learner or student is the primary focus of instruction.
- Interaction and "doing" are of primary importance
- Working in groups is an important mode of learning.
- Structured approaches to developing solutions to real-world problems should be incorporated into learning.

The benefits of collaborative learning include:

- Development of higher-level thinking, oral communication, self-management, and leadership skills.
- Promotion of student-faculty interaction.

Cooperative Learning

This is a mode of learning where the learners help each other to learn. Those who have better knowledge and experience help other students. It must be ensured that the help involves not a mere copying of the work of the other students. It is a need based interaction providing support for learning at all stages. All the members of the group will be ready to answer the questions on the common task and the group's achievement will be evaluated on the basis of the performances of the members of the group.

Why Use Cooperative Learning?

- Students who engage in cooperative learning learn significantly more, remember it longer, and develop better critical-thinking skills than their counterparts in traditional lecture classes.

- Students enjoy cooperative learning more than traditional lecture classes, so they are more likely to attend classes and finish the course.

Similarities and Differences between Cooperative and Collaborative Learning
Similarities:

1. Stress the importance of active learning.
2. The teacher acts as facilitator
3. Teaching and learning are experiences shared by both the student and the teacher.
4. Enhance higher order cognitive skills.
5. Greater emphasis is placed on students' responsibility for taking charge of her or his learning.
6. Involve situations where students must articulate ideas in small groups.
7. Help students develop social and teambuilding skills.
8. Increase student success and information retention and Utilize student diversity.

Differences

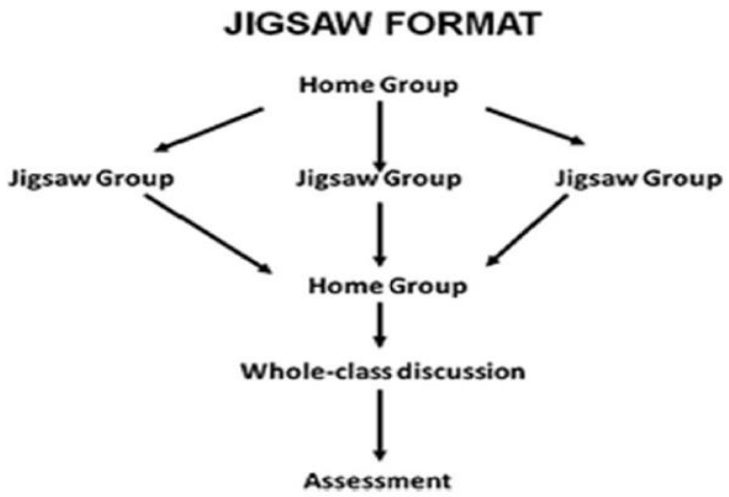
Cooperative	**Collaborative**
1. Students receive training in small group social skills.	**1.** There is the belief that students already have the necessary social skills, and that they will build on their existing skills in order to reach their goals.
2. Activities are structured with each student having a specific role.	**2.** Students organize and negotiate efforts themselves.
3. Activities are structured with each student having a specific role.	**3.** The activity is not monitored by the instructor. When questions are directed towards the teacher, the teacher guides the students to the information needed.
4. Students submit work at the end of class for evaluation.	**4.** Students retain drafts to complete further work.
5. Students assess individual and group performance.	**5.** Students assess individual and group performance.

Learning Together

The learning together method is a technique developed by D.W. Johnson and R.T. Johnson. The most important features of this technique are the existence of the group goal and sharing the opinion and materials, division of labour and the group reward. During the first applications to put out a single product working in groups, sharing ideas and materials, asking each other their questions before teacher have supplied to be rewarded.

Jigsaw method:

Jigsaw : Groups with 4/5/6 students are set up. Each group member is assigned some unique material to learn and become an "expert" on some aspect of a unit of study.



After reading about their area of expertise, theexperts from different groups meet to discuss their topic, and then return to their groups and take turns teaching their topics to their group mates.

Example:

Std : IX Topic: Measures Of Central Tendency

Learning Task :The students will be able to solve 2 problems that would require them to find the mean, median, mode and range from the given data.

Divide students into 4 member jigsaw groups.

The groups should be diverse in terms of gender, ethnicity, race, and ability.

Divide the lesson into 4 segments:

1. Solving for the mean from grouped, ungrouped data

2.Solving for median from the grouped, ungrouped data

3. Solving for the mode of the grouped, ungrouped data

4. Finding the range of the given data.

Assign each student to learn one segment, making sure students have direct access only to their ownsegment.

Form temporary "expert groups" by having one student from each jigsaw group join other students assigned to the same segment.

Give students in these expert groups time todiscuss the main points of their segment and to rehearse the presentations they will maketo their jigsaw group.

Bring the students back into their jigsaw groups. Ask each student to present her or his segment to the group. Encourage others in the group to ask questions for clarification.

At the end of the session, give a quiz or test on the material.

10.1 Test, Examination, Measurement, Assessment and Evaluation

In layman point of view, these words are often used more or less in same meaning (for example, test, examination) and of course all of them seem related to the process of judgement. Let us consider each of them separately.

Test and examination are used many a times interchangeably, or test is considered a mini examination, for example, monthly test/unit test/slip test and annual examination. But then 'TET' (Teacher Eligibility Test), that you will take after completing your B.Ed. programme or 'Admission Test,' you successfully cleared for getting into the programme, are also tests, which by any stretch of imagination cannot be called mini examinations.

For better understanding of these terms, let us see another example in day to day life: Suppose, you go to a doctor. Will s/he examine you or test you? Certainly, here the two words cannot be used interchangeably, and we say that doctor examines a patient and may prescribe certain tests. Thus, **test** is basically a tool (for example, intelligence test, aptitude test, achievement test, etc.) and **examination** is the process. **Measurement** as we are all familiar, is assigning to numerical value and the unit (if need be) in which a quantity is measured, i.e. 5 cm, 10 kg or 15g, etc. Let us now go back to our earlier example to understand the other two words, 'Assessment' and 'Evaluation'. After you get the tests conducted in a laboratory, you get a report which contains measurement(s) (numerals + units) of tested parameters. For example, your haemoglobin value is 16 gm/dl (deciliter). Along with these, there is another column in the report, which assigns certain meaning to these measurements, i.e. below normal, normal, above normal, etc. **thus, assigning meaning to a measurement is called assessment, which may be quantitative as well as qualitative, and final conclusion drawn on the basis of several assessments along with value judgement is called evaluation.**

- Test is used to gather information (a tool)
- That information is presented in the form of measurement
- That measurement is then used to make evaluation.

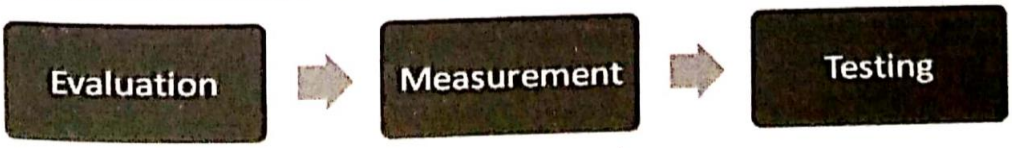


Fig.-10.2: Evaluation

Evaluation has wider meaning. It goes beyond measurement. When from useful information including measurement, we make a judgement that is evaluation.

Evaluation is a Scientific of providing information for decision making. It includes measurement, assessment and testing. It is a process that involves:

- Information gathering
- Judgement forming
- Information processing
- Decision making

Let us now see, **how assessment and evaluation are intertwined in classroom situations.**

i) **When the teacher starts any lesson,** the teacher needs to know the present status of knowledge of the students. To know this teacher may involve the learners in dialogue, conversation and inquiry. It helps to know about their prior knowledge. Varieties of learning opportunities are provided to know about their capabilities, interest and needs.

ii) **During teaching-learning process** the teacher needs to connect present knowledge with the previous knowledge. In this process what are their misconceptions and new concepts? Is there any modifications are required to improve the performance of the student as a learner and his/her own performance as a teacher? How is the class going on? How effective I have been using of approaches and strategies of teaching? In order to know all these, s/he again has to make some kind of evaluation. By asking questions s/he monitors process of their learning and concept development, provides continuous feedback and encourages them to reflect on their learning. The teacher facilitates them to construct and reconstruct the new knowledge.

iii) **At the end of the lesson/unit/session,** how can I know what I taught? To know this, the teacher has to make judgement of learning of the students and document their learning evidences. The teacher comes to know the extent of success of his/her teaching-learning processes, when the learner is passed and promoted to next class. Thus, s/he has to make a final assessment of the learners, for which s/he again has to take recourse to some kind of evaluation. Thus, we see that assessment and evaluation are ongoing processes. Therefore, the evaluation is the integral part of the teaching-learning process.

TYPES OF ASSESSMENT

Placement, Formative, Diagnostic and Summative:

Assessment is conducted in different phases of teaching-learning process. Some assessments are conducted before beginning of the teaching-learning process; some assessments are carried out during the teaching-learning process; and other kinds of assessment are conducted at the end of the instructional process.

Assessment can be classified into four types on the basis of their purposes:

1. Placement Assessment
2. Formative Assessment
3. Diagnostic Assessment
4. Summative

Assessment Each of four types of assessment serves different purposes. In the teaching learning process, it starts with placement assessment and ends with summative assessment. All four types of assessment are important, and are unique in their functions. The four types of assessment and their functions are presented in below the table for your understanding

Areas of Function	Types of Assessment and their functions
After Instructional Process	*Summative Assessment* (to certify the learner)
During Instructional Process	*Diagnostic Assessment* (to solve learning difficulties)
	Formative Assessment (to provide feedback on the teaching-learning process and to know mastery in content)
Before Instructional Process	*Placement Assessment* (to know entry behaviour)

Table represents different types of assessment which are conducted in the teaching-learning process. Generally, assessment starts with measuring the entry behaviour of learners to form judgement about their terminal behaviour. Let us discuss the main purpose and functions of different types of assessment:

1) Placement Assessment:

Placement evaluation is conducted before the organisation of **teaching-learning activities to measure the entry behaviour or previous knowledge of learners.** Another purpose is to know whether learner is able to acquire the new learning experience which

is related to the previous knowledge. The key word which is used for placement assessment is the "entry behaviour". Assessment of entry behaviour is done just before teaching starts. In the teaching-learning process, before teaching a new topic, a teacher should know the previous knowledge of students. This helps teacher to organise teaching-learning activities according to the previous knowledge of learners.

2) Formative Assessment:

For the first time in the year 1967, Michel Scriven used the concept of formative assessment in the field of curriculum evaluation. Scriven (1991) defined it as "Formative assessment is typically conducted during the development or improvement of a programme or product (or person, and so on) and it is conducted, often more than once, for in house staff of the programme with the extent to improve". If we analyse the definition, it is clear that the purpose of conducting **formative assessment is to monitor the learning progress of the learner; it is also conducted to know whether the learning objectives have been achieved or not and to provide feedback on the teaching-learning process.** The key word in formative evaluation is mastery of learning or learning progress. According to above table it is conducted during the instructional process. It is considered as the second stage of assessment which is conducted during the teaching-learning process. It is carried out from the very beginning of instruction and continues till the end of the course.

The examples of formative assessment are unit end examination, monthly examination, quarterly examination, half yearly examination, etc. It provides the teacher feedback about the progress of learners in the programme and about the effectiveness of the programme. It also provides data for diagnostic assessment. Let us discuss the diagnostic assessment.

3) Diagnostic Assessment:

It is the assessment which is conducted along with formative assessment during the instructional process. It is carried out based on the data obtained from formative assessment. **Diagnostic assessment is specially conducted for removing the learning difficulties of learner**. For example, if it is found that a learner has not understood certain concepts in a particular subject, then to help him/her understand these concepts, diagnostic assessment is conducted and remediation is provided. This is conducted by diagnostic remedial test. The key word in diagnostic assessment is assessment of learning difficulties. Diagnostic assessment not only solves learning difficulties of learners but also identifies and provides remedies for personal and psychological problems.

4) Summative Assessment:

Summative assessment is conducted to know the Assessment: Basics terminal behaviour of learner. The key word in summative assessment is "certification". **Summative assessment is conducted after completion of the whole course.** Feedback provided in summative assessment is terminal in nature and cannot be used for modification of learners' behaviour because it is conducted at the end of a term. Learners get certificate or are promoted to the higher class based on summative assessment. Different techniques and tools used in summative assessment are verbal or non-verbal tests, and teacher made or standardised tests.

EVALUATION AND MEASUREMENT IN MATHEMATICS

"Evaluation is relatively a new technical term, introduced to designate a more comprehensive concept of measurement than is implied in conventional test and examination."

—According to Encylopedia of Educational Research

The main and first point of teaching process is instructional objectives. When the teaching objectives are not decided we can not decide the direction of teaching. For achieving the teaching objectives teacher creates this type of situation, which are helpful to desirable behavioural change in teaching. For this purpose teacher should use the specific teaching aids, teaching methods etc. After this teacher evaluate the quantity of achievers objectives. Evaluation is that evidence which give the success or failure of teacher to his/her teaching objectives.

The effectiveness of instruction is usually determined by measuring achievements against objectives undertaken. An efficient programme of evaluation no longer comprises merely the effort to check the completed process but rather in the continual appraisal of the student's progress towards the attainment of preestablished aims. There is probably no more accurate barometer of the fundamental philosophy of any curriculum than a careful analysis of its evaluation programme.

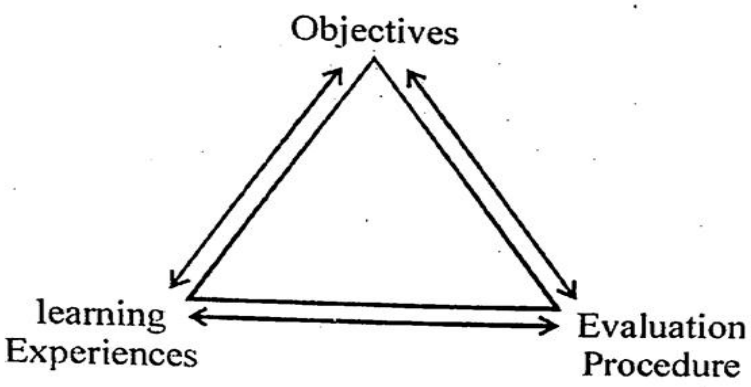
Concept of Evaluation

In the traditional examination system more emphasis is given by the content and its achievement by the students. According to concept of evaluation, the entire learning situations are responsible for the success and failure of students. Therefore, in the process of evaluation the effectiveness of teaching, method and techniques of teaching, teaching aids, curriculum and other learning situations are evaluated on the basis of achievement of learning objectives.

Meaning of Evaluation

Evaluation is defined as a process of collecting evidences of behavioural changes and judging the directions and extents of such

changes. This means that evaluation is free neither from instructional objectives nor from the teaching learning.



In fact, it is intimately related to objectives and learning activities on the one hand and improvement of instructions on the other. The evaluation procedures and learning experiences provided to children are the instruments or means related to these ends. These are interrelated to each other.

It is clear from the above triangular relationship that :

- All the three steps are inter-related.
- Evaluation is a natural step involved in teaching-learning process.
- The objectives of teaching constitute a pivotal step on which both learning experiences and evaluation techniques are based.
- Evaluation based on objectives of teaching and learning experiences evaluates the objectives also at the same time.

Hence, in classroom, teacher organizes suitable teaching learning activities in order to promote desired expected changes in behaviour. The learning experiences offered in class-room are directly related to the predetermined in functional objectives. Thus, the teaching-learning activities are objectives based not content or text-book based.

The evaluation may be defined as follows :

Evaluation ↔ Measurement + Value Judgement

Here, evaluation is both quantitative and qualitative process.

Measurement	Value Judgement
↓	↓
quantitative	qualitative

Definitions of Evaluation

Evaluation is a broad and goal oriented process. In the field of education the main work of this to make the education is goal or object centred. On the basis of this we can achieve the knowledge of development

of whole personality. Hence different educationist define the evaluation in many ways :

According to **Rammere and Gage,** "Evaluation assumes a purpose or an idea of what is 'good' or 'desirable' from the stand point of the individual or society or both."

According to **Vasely,** "Evaluation is the inclusive concept, it indicates all kinds of means to ascertain the quality, value and effectiveness of desired outcomes. It is a compound of objective evidences and subjective observation. It is the total and final estimate. It is valuable and indispensable guide to the modification of policies and to further action."

According to **Garrett,** "Evaluation is the group of that type of examination question, which are prepared for the treatment of any skill or ability."

According to **Kothari Commission,** "It is now agreed that, evaluation is a continuous process, forms an integral part of the total system of education and is closely related with educational objectives."

According to **T.L. Torgerson,** "To evaluate is to ascertain the value of some process or thing. Thus, educational evaluation is the passing of judgement on the degree of worth whileness of some teaching process or learning experience."

According to **Dandekar,** "Evaluation may be defined as a systematic process of determing extent to which educational objectives are achieved by pupils."

According to **Quillen and Hanna,** "Evaluation is the process of gathering and interpreting evidence on changes in the behaviour of the students as they progress through school."

According to **James M.Lee,** "Evaluation is the appraisal of pupil's progress in attaining the educational goals set by the school, the class and himself. The chief purpose of evaluation is to guide and further the students, learning. Evaluation is thus a positive rather than a negative process."

According to **Moffat,** "Evaluation is a continuous process and is concerne with more than the formal academic achievement of students. It is interested in the development of the individual in terms of desirable behavioural changes in relation to his feeling, thinking and actions."

On the basis of above definitions we can say that, evaluation is a continuous process, forms an integral part of the total system of education and is intimately related to educational objectives. It exercises a great influence on the pupil's study habits and the teacher's methods of

instruction and thus helps not only to measures educational achievement but also it improves it. Hence

- Evaluation is both qualitative and quantitative.
- Evaluation is a broad concept.
- It is a continuous process.
- It is the appraisal of student's progress, in attaining the educational goals set by the school, class and himself.
- Teaching-Learning process becomes dynamic and self-developing.

Factors of Evaluation

Two processes have to be undertaken in evaluation

(i) the measurement

(ii) analysis of the information or data obtained from measurement.

We aware that there are the following four factors of measurement:

(a) The object, person or activity any of which characteristics has to be measured.

(b) The characteristic of the object, person or process which has to be measured.

(c) The tools and devices of measuring such characteristic.

(d) The person who measures it.

Need of Evaluation

Basic needs of evaluation are as follows :

(a) Evaluation gives guidance to the teacher.

(b) It is necessary in the field of mathematics to classify the students on the basis of their achievement.

(c) It helps to make the body and mind disciplined.

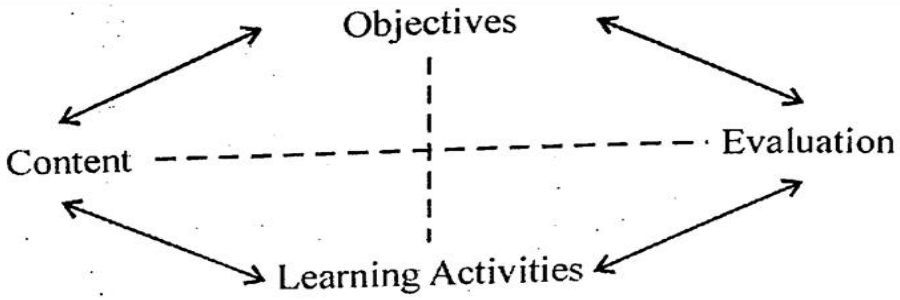
(d) It clarifies the objectives of teaching.

(e) Evaluation helps the teacher to identify the difficulties, problems and weakness of the students.

(f) It stimulates the students and gives them the inspiration to study.

Steps of Evaluation Process

Evaluation is a continuous process, which continues to birth and ends to his/her death. The concept of evaluation can clarify the objectives, content, learning activities and evaluation techniques and interrelationship between them.



Objectives, content, learning-experiences and evaluation techniques etc. are included into evaluation process. So these four are inter-related to each other. Achieving the objectives are very essential and it achieves directly whenever the content, learning-experience are jointed to indirect form. The steps of evaluation process in education is :

1. Formulation and definition of educational objectives
2. Planning Learning Experiences
3. Evaluating on the basis of behavioural change

1. Formulation and Definition of Educational Objectives–For this purpose we firstly determine the objectives, which are a typical job. After having determined it, he gives it a definite form and defines it. For example, if he has to provide educational guidance, he has to determine the traits that he has to measure, such as intelligence, interest, aptitude, attitude etc. In case he has to measure the educational achievements, then he has to determine the aims and objectives that he has kept in mind while teaching a subject or training in an activity and he has to determine the extent to which he has to achieve it.

2. Planning Learning Experiences–"Learning occurs when pupil has an experience, that is, when he reacts to the situation in which he finds himself. Hence, learning experiences are the means to attain educational goals." ***—The Concept of Evaluation in Education, NCERT***

The meaning of planning of learning is the construction of that aids, which are helpful to develop desirable behaviour in the child. For this purpose teacher should create that situation which activates the child. Teacher should keep in mind the educational objectives, age, maturity, child's behaviour etc. of the child, which are useful or helpful to create or develop a new knowledge in the child.

3. Evaluating on the basis of behavioural changes–The main and last objectives of education is the desirable change in the child's behaviour. This change is due to come about the teaching process which gives the knowledge of cognitive, affective and psychomotor domain to the child. All these domain are co-related to each other. These behaviours are included in external or internal behaviour the child.

Basic Assumption of Evaluation

The main assumptions of evaluation are as follows :

1. Human nature is complex so in that way the measurement of different dimensions are necessary to their evaluation.
2. The work of education is to change to the desired behaviour of child so evaluation is important.
3. The achievement level of objectives can be measured by only evaluation.
4. Evaluation can be preceded by teachers, parents and other persons.
5. Evaluation theories and moral values should be followed in correct ways.
6. Measurement and evaluating decision both are included in evaluation.

Purpose of Evaluation

The main purposes of evaluation in teaching process are :

1. The main purpose of this to evaluate the students than classified it.
2. Evaluation is helpful to change to desirable behaviour of the child.
3. With the help of this modification in curriculum can be possible.
4. Evaluation decides the limit or level of student's objectives.
5. Evaluation measures the purity of teaching methods.
6. Evaluation is helpful to understand the student's problems.
7. It is also helpful to knowing the progress of the students.
8. By the help of evaluation gives the proper direction or guideline of educational and vocational guidance.

Functions of Evaluation

The following are the main functions of evaluation :

1. It is helpful to knowing the ability, capability, interest etc. of the students.
2. It ascertains that how far could learning objectives be achieved.
3. The work of evaluation is the change and improvement in apparatus.
4. It improves in curriculum.
5. It diagnoses the weakness of instructional procedures and provides the basis for remedy.
6. It is helpful to all round development of the child.
7. It gives reinforcement and feedback to teacher and students.
8. It helps in developing a comprehensive criterion test.
9. It provides the empirical evidences about the effectiveness of teaching strategies, tactics and aids and suggests some modification improvement.

Importance of Evaluation

The importance of evaluation are as follows :

1. Knowledge about the relation of ability in the child–The first and important work of education is to develop the mental, physical and emotional development of the child. For the mental development develop the knowledge, understanding, application, analysis, synthesis etc. in the child. Due to lack of this the mental development is not possible.

2. Knowledge is related to the achieving of the goals–The knowledge of achieving goals can be measured by evaluation. Evaluation is a continuous process. It goes hand in hand with the teaching-learning process. Evaluation implies a systematic process which omits the casual uncontrolled observation of pupils or students.

3. Knowing to the relation achievement of students–Evaluation process is helpful to knowing the student's achievement. This opportunity is found only at examination time. At this time student expresses his knowledge in the form of oral or written test. Evaluation is more concerned with the growth which the student has made, than with his status in the group.

4. To know the knowledge of data limits–The knowledge of data limits is essential to give the learning experience. Development of child depends upon these learning experiences. On the basis of this we conclude that the limit of abstract development empowers in the student.

Characteristics of a Good Evaluation Programme

The main characteristics are :

1. It measures the direction and extent behavioural changes.
2. Evaluation is quantitative and qualitative estimation of specific changes in children's behaviour.
3. Evaluation is diagnostic so that it may provide basis for remedial teaching.
4. Evaluation is pupil-oriented.
5. Evaluation offers feedback to the entire educational system.
6. It is activity based.
7. It may be used to improve instructions, curriculum, methods and examinations etc.

MEASUREMENTS

Measurement is related to human race right from the drawn of civilization. Due to lack of measurement imagination of scientific progress can not be measured. Man's innate curiosity motivated him to make an attempt to know his surroundings and to define or describe these attributes.

Meaning of Measurement

In simple language, to measure and show the weight, length and volume of an object in a definite units is called measurement. for example– To show the weight of a person in kilograms, length of cloth in metres, volume of oil or milk in liters. But the field of measurement is very wide. Measurement is very essential in the study of science. Science is collection of information and observation about the natural or physical phenomena. Mathematics is a universal language which can be easily utilized by any science, its technology is unlimited yet well definite. It is easily possible to communicate, correctly and objectively through measurement.

For each measurement two type of knowledge is essential :

(i) Which quality or characteristics are measured.

(ii) Tool of measurement.

Definitions of Measurement

According to **Thorndike,** "Anything that exists at all, exists in some quantity, anything that exists in some quantity is capable of being measured."

According to **S.S. Stevens,** "Measurement is the process of assigning numbers to objects according to certain agreed rules."

According to **Gilford,** "Measurement means the description of data in terms of number."

According to **E.A. Peal,** "It's purpose to promote the development of a well integrated person, capable of exercising such responsibilities in society as is power allowed."

According to **Campbell,** "Assignment of numerals to objects or events according to certain rules is called measurement."

According to **James M. Bradfield,** "Measurement is the process of assigning symbols to the dimensions of phenomenon in order to characterize the status of phenomenon as precisely as possible."

In these definitions of measurement only the qualities of objects and activities have been included, and not the measurement of qualities of persons. Though the persons are included in the objects of the universe, however, the objects are meant to include only concrete materials, so it is necessary to show the measurement of qualities of the persons separately.

Measurement is a process of quantification. It means precision and quantification of a phenomenon or variable, but not a person or object.

Characteristics of Educational Measurement

The main characteristics of educational measurement are :

1. The units of measurement is not fixed.

2. There is no any absolute is zero point.
3. Measurement is based on proper direction but not direct.
4. Measurement is helpful to evaluate the person.
5. Measurement is more economical than subjective evaluation.

Limitations of Measurement

Some limitations of measurement are :

1. The area of measurement is narrow and fix.
2. It gives only information, not decisions.
3. The process of measurement is very complicated.
4. In the measurement, we measure those traits, which are abstract and very small or micro. By which the measurement is not correct.

Essentials of Measurement

Measurement in any field always involves three essentials :

1. Identifying and defining the quality, attribute or variable that is to be measured.
2. Determining the set of operations by which the attribute of variable may be made manifest and perceivable.
3. Establishing a set of procedure of definitions for translating observations into quantitative statement of degree, extent or amount.

Scales of Measurement

Measurement involves quantifications and assigning of numerals. **Stevens** has defined that measurement is a process of assigning symbols or numerals to the objects according to certain agreed rules.

According to **Helmstadha,** "Measurement is the person or thing processes some characteristics. A convenient scheme for classifying measurement scales according to the :

- Levels of refinement
- Degree of accuracy
- Complexity
- Sophistication

There are four scales of measurement known as:

- Nominal scale
- Ordinal Scale
- Interval Scale
- Ratio Scale

It is common to distinguish four levels or scales of measurement, which have been most clearly delineated by Stevens. From lower to higher levels we have nominal, ordinal, interval and ratio scale of measurement.

Difference between Measurement and Evaluation

Generally people understand measurement and evaluation as equal but actually they are different to each other. Measurement is the first step of the evaluation process. Evaluation includes analysis of results after measurement and prediction is made on the basis of this analysis.

According to **Wrighstone,** "The emphasis in measurement is upon single aspect of subject-matter achievement or specific skills and abilities but ..... the emphasis in evaluation is upon broad personality changes and major objectives of an educational programme. These included not only subject-matter achievement but also attitudes, interests, ideals, ways of thinking, work-habits and personal and social adaptability e.g. by testing a child in mathematics we may measure his mathematical ability and nothing else. We may not have any idea about the interests, abilities etc. of the child in maths by administering this single test."

In brief, we can say that, Measurement is quantitative while evaluation is qualitative.

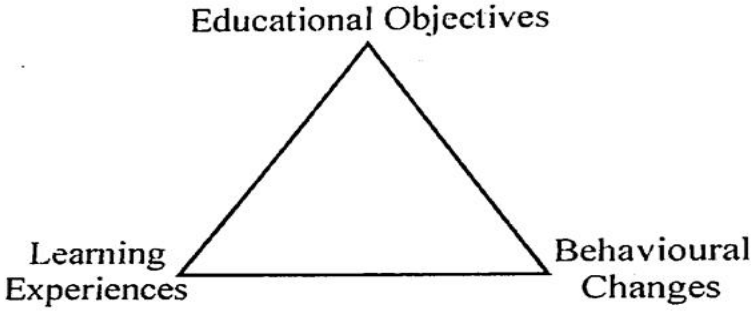
The differences between measurement and evaluation are as follows:

Measurement	Evaluation
1. Measurement is an old concept	1. Evaluation is a new concept.
2. It is narrow and less inclusive concept than evaluation.	2. It is wider and more inclusive concept.
3. It is a normal word.	3. It is a technical word.
4. It is quantitative process.	4. It is a quantitative as well as qualitative process.
5. It is a part of the evaluation process.	5. It is a continuous process.
6. Evidences are collected in the measurement process.	6. Evidences are analysed under the evaluation process.
7. There are four steps in this process.	7. There are six steps in this process.
8. Teaching-learning process can not be improved and modified.	8. The teaching-learning process can be improved and modified.
9. It is a short process.	9. It is a lengthy process.
10. Measurement is the first step of evaluation.	10. Under evaluation, the results are also analysed after having obtained result from measurement.
11. They can not be properly classified on the basis of results obtained from measurement.	11. They can be properly classified on the basis of results obtained from evaluation.

12. Only cognitive domain objectives can be measured. 13. Comparative study is not possible.	12. Objectives of all the three domain can be evaluated. 13. Comparative study is possible.

New Concept of Educational Evaluation

Educational objectives, learning experiences and behavioural changes are related to education process. In this firstly decided the educational objectives and then according to this objective we achieve the learning-experience, which are helpful to desirable behaviour change in the students. It is a triangle process bloom represent this with the help of graphic representation which are as :



New concept in educational evaluational give more emphasis in the following points :

1. The centre point of this is child not the subject-matter.
2. It gives more emphasis on learning than teaching.
3. It caters to the pupil's psychological needs, interests, ways of thinking, reasoning, personal and social ability of adaptation.

On the basis of above three points the educational evaluation can be defined in a following ways:

"Evaluation is any systematic, continuous process of determining– (a) The extent to which the specified educational objectives previously identified and defined are attained, (b) The effectiveness of the learning experiences provided in the class-room and (c) How well the goal of education has been accomplished."

On the basis of this it can be said that:

(i) Evaluation is a continuous process.

(ii) Evaluation is a systematic process. It omits the casual uncontrolled observation of the pupils.

(iii) Evaluation is not related to the subject matter achievement but it gives an important place in ideas, interests, capabilities, needs and reasoning.

(iv) Evaluation always assumes that the objectives are predetermined and defined.

(v) Evaluation gives more emphasis to learning than teaching.

Testing

The origin of psychological test is individual difference with psychology. Firstly it gives **Isqural** and **Saguine**. And Further **Caber, Frakner, Mullar, Alexjender Wage** work in this. In general language test is that aid or material to study the human's behaviour which are helpful to understand it.

According to **Cronbak,** "A test is a systematic procedurce for comparing the behaviour of two or more persons."

According to **Freeman,** "A psychological test is a standardized instrument designed to measure objectively one or more aspects of total personality by means of verbal or non-verbal responses or by means of other behaviour."

Difference between Evaluation and Test

In ancient period it comes. Today it is not related to education but also it is related to every field of life. Some disimilarties in evaluation and test are given below :

Evaluation	Test
1. Evaluation is a formal and informal process.	1. Test is only formal process.
2. It is a modern concept.	2. It is an old process.
3. It is a broad and lengthy process.	3. It is narrow process in comparison to evaluation.
4. It is a objective-centred.	4. It is content-centered.
5. It is a continuous process.	5. It is not.
6. It evaluates the cognitive, affective and psychomotor domain in the students.	6. Test evaluates only cognitive domain of the students.
7. Evaluation is less objective and reliable.	7. It is more objective and reliable in comparison to evaluation.
8. Result of evaluation is qualitatives and quantitative both.	8. The result are only quantitative.
9. It evaluate only broad quality.	9. It measures the specific ability of the child.

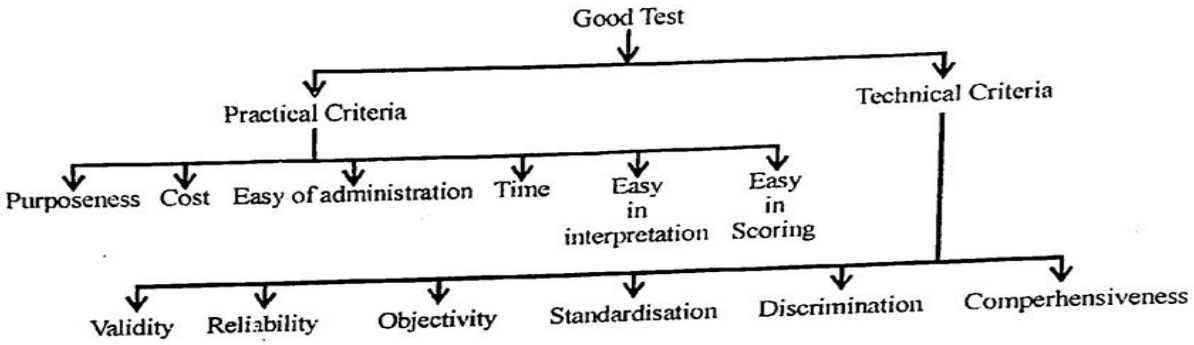
Characteristics of a Good Test

"A test is a systematic procedure for comparing the behaviour of tw or more persons."

According to **Klausmeier and Goodwin,** "Good standardized test must meet the criteria of validity, reliability and usability."

A good test can be classified in to two categories:

(i) Practical Criteria
(ii) Technical Criteria



(1) Practical Criteria

They are following type :

(i) Purposeness–It is essential to any test that what is the purpose of that test. Construction of any test is depend upon their purposeness. It is clear in itself. For example exam of mathematics is not the measure of the knowledge of english.

(ii) Cost–A good test should be economical. At the time of test construction it should be kept in mind that test is not high cost to a researcher. For this purpose only those steps should be taken in the test which are easily found the objectives.

(iii) Easy in Administration–"A good examination must possess a number of characteristics and these characteristics become the basic principles underlying the construction of each test."

—Douglas and Holland

A good test is that they easy to given answer, the situation of that given answer are easy, instructions are easy and clear and the scoring is objective.

(iv) Easy in Interpretation–Any test should be easy at the point of lecture or interpretation. A good test is that which is interpretated easily in the class by the teacher. For this purpose teacher does not use the high statistical formulas.

(v) Clearness–The direction given should be brief and definite so that children may avoid wrong performance due to miss understanding of

direction. The language of the items should be simple understandable and definite.

(vi) Easy in Scoring–A test should br easy at the scoring point of view. For this purpose objective type questions should be prepared because it is easy for scoring purpose.

(vii) Acceptability–Acceptability means, to administrated that test to that persons and that situations successfully which is useful to process of standardization of that test. In a good test the question is not very easy and nor tough.

According to **Frederic G. Brown,** "A test is a representative when the test items are similar to the behaviour, we are interested in measuring."

(2) Technical Criteria

(i) Validity—A test is said to be valid if it actually measures what is supposed to be measured. Validity is a truthfulness or purposiveness of a test. It is essential quality of any test. The test in the field of education are generally constructed to measure some specific qualities or abilities of the students. If a test measures the qualities or abilities for which it was constructed, then such a test is called valid and this quality of the test is called validity.

For example : we take an achievement test. If this test measures the knowledge, skill and sentiments for which it was made, then it will be called a valid test and its this quality will be called validity.

"In general, the term validity refers to the effectiveness of a test in achieving specified purposes." —***Stodala and Stordall***

"A valid test is one which measures the traits and qualities it is intended to measure." —***Kolesnik***

So, validity is an important characteristic of a good test. It is the extent to which a test measures what it purports to measure.

(ii) Reliability—Reliability implies accuracy and consistency in behaviour. In the field of education, the tests are constructed for the measurement of various qualities and abilities of the students. If a test measurable qualities correctly and there is no possibility of a measurable error then, such a test is called a reliable test, and this quality of test is called reliability.

Reliability = Sample size + Objectivity

For example: If we administer standardised intelligence test on a group of students today, and then administer it again after 10-20 days on the same group and the measurement results are identical, such a test will be called a reliable test and this quality is called reliability.

"Reliability refers to the faith that may be placed into a test."
—***Rizeland***

"Consistency thou art of the jewel." —***Sexpeyear***

"A test is reliable, if it measures consistently." —***Skinner***

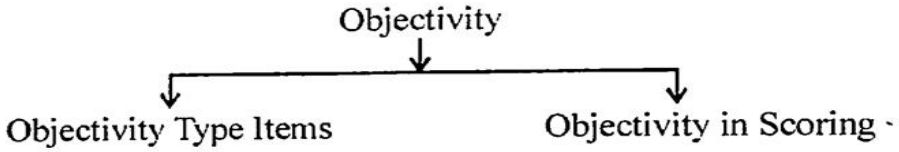
"Reliability-always refers to consistency throughout a series of measurement." —***L. J. Cronbah***

Hence, if a test measures the measurable qualities correctly for which it was constructed, this quality of the test is called reliability of the test.

(iii) Objectivity—The test whose measurement results are not influenced by personal proclivities or opinions as-like, dislike etc. of the measurer and the results are similar whoever the measurer might be, is called an objective test and this quality of the test is called objectivity. It is an important characteristic of a good test that affects both reliability and validity of the test. It means that the personal assumptions and pre-thoughts of the examiner should not affect the scores of the child.

Objectivity has two aspects:

(a) Objectivity type items
(b) Objectivity in scoring



(a) Objectivity Type Items–Items in the test should be as simple as possible. The pupil should be able to interprete the item correctly. These should not be influence on pupils mental status.

(b) Objectivity in Scoring–The variations in the examiner's mood, feeling of examiner, his attitudes and prejudices should not affect the scoring of the test.

"A perfectly objective measuring instrument must yield the same measurements or scores in the hands of all competent persons."—***Ruseland***

(iv) Standardisation–A standardized test is that test, in which procedure, apparatus and scoring are definite. It can be given different time and place at appropriate method or way.

"A standardised test is that in which the selection of content on the basis of experiences, the development of administration and scoring of equal method and the scoring has been done by objective method."
—***C. V. Good***

(v) Discrimination–A good test must pick out good and poor students from a population or a group. Each item of the test must discriminate between good and poor children. If an item is answered by an equal number

of intelligent and poor student it has no discrimination. But if only intelligent children are able to do an item and most of the children miss it it has highly discriminating value. After the administration and scoring of the test, each item of the test is analysed to calculate the level of difficulty.

(vi) Comprehensiveness–A test is comprehensive to the degree to which it samples sufficiently widely into the subject. It is said that a test should be comprehensive enough to be valid. It should include all units, sub-units and topics of the test content. Hence it should cover the whole syllabus.

(vii) Difficulty Level–Difficulty level is an important character of a question. Whole questions in a question paper can be divided on the basis of difficulty level. A good test consists of 66% normal difficult questions, 17% high difficulty level and 17% low difficulty level questions.

(viii) Usability–Those test while is easy to construction, easy to given answer and score of students, called a good test. In other words a good test is that which is convenient and practicable for both the teacher and student. A good test should be easy, attractive and interesting. Usability means no difficulty should come across which using a particular test.

"By usability we mean the degree to which the test or other instrument can be successfully employed by teachers and school administration without any undue expenditure of time and energy. In a word, usability means practicability." ***—C. C. Ross***

(ix) Norms–Norms are that average performance of the group in a test and it is based on the average achievement or capacity of the whole group. Norms indicate the present achievement. Generally norms of a test are determined on the basis of obtained scores of the test. These are the abbreviated form of obtained scores.

"Norms of a test describe how the members of a specific referenced group obtain marks in a test." ***—Ebel***

"A norm is the performance level got in a test by a defined group of students." ***—Remmers, Gage and Rummel***

Norms are calculated in the form of age, grade, percentile and standard score. It is an important characteristic of a good test.

General Principles of Test Construction

General principles of test construction are :

1. Construction of test should be comprehensive.
2. The language of question should be simple, clear and short in a test.
3. Bised or two way meaning question should not be include in a test.
4. The test should be reliable and valid.
5. The quantity of question is appropriate in a test.

6. Arrangement of questions in a systematic ways.
7. Nomenclature of a test should be proper.
8. General instruction related to test should be given in the starting of the test.
9. Construct those questions which can evaluate the capacity, ability of the students.

Types of Test

Generally test are of two type :

1. Teacher made test
2. Standardized test

(1) Teacher-made Test

These are tests prepared by the teachers themselves. Reorganised general objectives are the main things that help in the preparation of these tests. These tests are quite refined and objectives. Procedure of these tests is similar to standardised test to a great limit. These test are objective but not standardized.

Objectives of Teacher-made Test

The main objectives to construction of teacher-made test are:

1. With the help of these test teacher should evaluate the content successfully.
2. These tests are more important at the point of continuous evaluation.
3. These tests can prepare easily in a short time period.
4. The main objective of these tests to improve the demerit of essay type test.
5. These tests give opportunity to make the effective teaching of himself.
6. For the construction of this test teacher is not need of special training.

Characteristics of Teacher-made Test

The main characteristics are as follows :

1. The questions of these tests in an objective form.
2. These tests can construct to any subject teacher.
3. These tests are not standardized.
4. In these tests the questions are short and more in quantity.
5. This test present the whole curriculum.
6. In this type of test we can collect the different type of forms easily.
7. This test evaluates both teacher and student.

Limitations of Teacher-made Test

Besides of above characteristics it contains some limitations :

1. By these test difficult the study of whole personally.
2. Impossible to complete knowledge about the subject.
3. Construction of these tests is a difficult work.
4. This test is difficult to knowing the student's difficulties.
5. These tests are overloaded to the teacher.

(2) Standardized Test

The word standardization means the process of finding comparative norms and the literary meaning is "to bring to a level or standard" standardization of test involves the critical analysation of curriculum, careful selection to items and strict observation of standards. Intelligence test, Interest test, Aptitude test, Achievement test and Personality test etc. are the standardized tests.

"A standardised test is one in which the procedure apparatus and scoring have fixed so that precisely the same test can be given at different times and places." ***—Lee J. Combach***

Purposes or Functions of Standardized Test

The main purpose of standardized test are :

1. To diagnose learning difficulties.
2. To plan instructions activities.
3. To group students for instruction.
4. To evaluate the curriculum.
5. To appraise gains in achievement of groups.
6. To consult parents and teachers.

Utility and Importance of Standardized Test

For the study of human standardized test have some utility and importance. Some are these :

1. Standardized tests are objective and unsided.
2. It takes very less time to collect the information.
3. By this test we can find the personality and behaviour related factors directly.
4. This test is helpful to knowing the difficulty of students.

Demerits of Standardized Test

Demerits of standardized test are as given below :

1. Detail measurement is not possible by this test.

2. Only measurement process is continuous by this test.
3. It is only a part of guidance programme of work not a whole work.
4. Sometimes standardized tests are used that objectives whose are not useful for that.

Difference between Teacher-made Test and Standardized Test

Teacher-made Test	Standardiged Test
1. Lack of reliability in this test.	1. In this more reliability.
2. Teacher-made tests are concerned with the limited and specific field of knowledge of performance.	2. Standardized tests are concerned with the whole field of knowledge or ability tested.
3. This test is constructed by the teacher.	3. This test is constructed by scientist.
4. It take less time to construction.	4. It take more time in construction.
5. No much norms or standards are provided in the teacher made informal tests.	5. In standardized tests norms are given for various groups of persons on age, grade, rurality, sex or other basis.
6. It is easy to construction.	6. It is typical or tough in construction.
7. Validity of these tests is less	7. Validity to these test is more.
8. Analysis of the terms of these test is made later they used.	8. Analysis of the terms of these tests is made earlier they used.
9. It is related to broad knowledge.	9. It is related to specific content.
10. It may just be made on the basis of the personal experiences of the teacher.	10. It is use of sources such as the opinion of judge, articles, general books is made.
11. Teacher made tests are used to know whether the student has attained knowledge in specific fields.	11. Standardized tests are useful in comparing achievement of individual groups.
12. Teacher made informal objective tests are for local use for some particular institutions on small scale.	12. Standardized tests are based on uniform curriculum in many schools in the prariner or the whole nations.

Difference between Evaluation and Examination

Evaluation	Examination
1. Evaluation is a modern concept.	1. Examination is and old concept.
2. The field of evaluation is broad.	2. The field of examination is narrow in comparison to evaluation.
3. Evaluation is a technique.	3. Exam is one method in many methods of evaluation.
4. Evaluation is used oral and written both type of test.	4. Generally exam word is used only written type.

TECHNIQUES OF EVALUATION

The meaning of evaluation techniques is that by which we can evaluate the knowledge and desirable changes in behaviour in the students and it also evaluate the characteristics according to individual differences. All measurement some sort of instruments are needed. A tool may be defined as an implement which facilitates the work of hand and eye. Evaluation devices are for both tool and techniques. He has pointed out that in the evaluation of achievement of any subject, tools and techniques are needed to facilitate measuring and recording the characteristics of pupils. These techniques should be objective, reliable, practicable, comprehensive and valid. Each behaviour may need a different type of device.

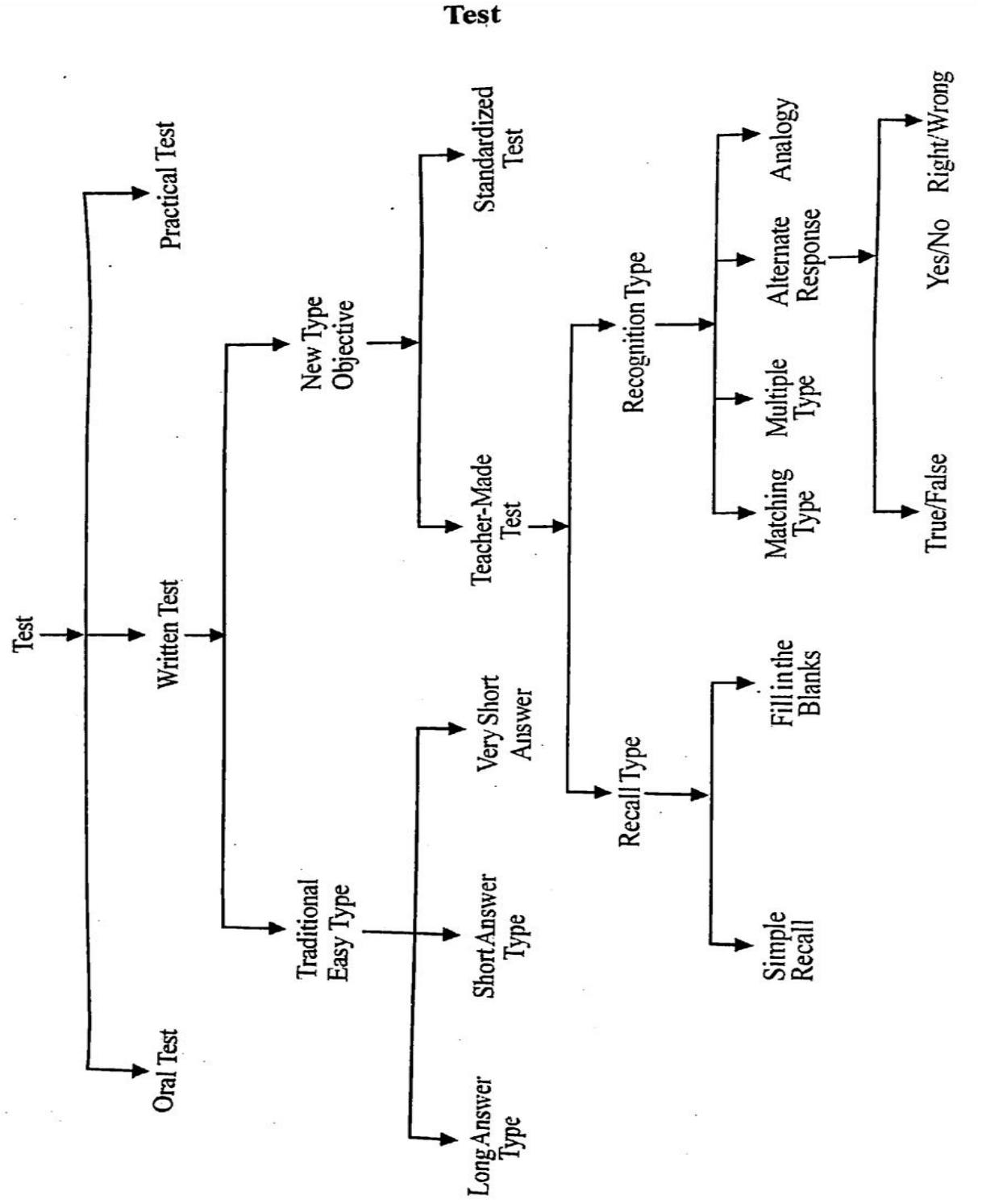
1. Oral Examination–In this technique oral questions, discussions and debates etc. are used. Questions are presented orally and answers are received in the same way. It is important for development of word-power and reasoning power. A larger number of students can be examined in a shorter time.

The aim of teaching of subject is not limited to impart knowledge of the related facts, it is also taught to bring about a change in the thinking and working style of the students. And its measurement is done by oral tests. These tests are used for ascertaining their knowledge and practical use of linguistic abilities, their confidence and their knowledge of other subjects.

Generally these exams are to evaluate the achievement level of the students, which are not possible by written exam. These exams are used to measure the recall, thinking, recognition power etc. of the students.

2. Practical Exam or Test–In this technique, some practical or hand work has to be performed by the student. This technique is useful in the evaluation of some topic of mathematics. The tests in which the measurement of skills and activities of the students is executed by asking them to display the skill or activity are called practical tests.

The chief characteristics of practical tests is that practical aspect of the examinees is measured. These tests alone can test the practical knowledge of arts and science. The practical skills can be tested only by practical tests.


Test
Test
Practical Test
Written Test
Oral Test
New Type Objective
Traditional Easy Type
Standardized Test
Teacher-Made Test
Very Short Answer
Short Answer Type
Long Answer Type
Recognition Type
Recall Type
Alternate Response
Multiple Type
Matching Type
Analogy
Fill in the Blanks
Simple Recall
Yes/No
Right/Wrong
True/False

3. Written Exam or Test–In this technique questions are presented in written form and their answers have to be written by the students. This test is used for the measure, the thinking power, language etc. of the students. They are of two type—

(i) Essay Type Test
(ii) Objective Type Test

(i) Essay Type Test

These type of examinations are being conducted since the ancient times. About 200 year ago, this method was utilized in selecting the rights in China. In 1854, Horack Mann, experimented this type of examination or test in America. This technique is subjective. It emphasises on roting power.

Eassy type techniques means that examination in which children have to write answers of some questions in fixed time. The construction of these questions is of such type that their answers take the form of essay. So it is called essay type examinations. The construction and administration of essay type test is simple and easy. It is economical technique.

In these tests some questions (8-10) are set from the whole syllabus and students are asked to answer any five questions. In these tests it is expected that students will present an explicit answer of every question. In essay type test student is free to express his views through his answers.

Types of Essay Type Test

Essay type tests are of following types :

(a) Organisational and Comparative–In this type of test comparison type, arrange in systematic order, organize type questions are included.

(b) Establish Relationship–Why? What? etc. type questions are include in this type of test.

(c) Selective Recall of Facts–Arrange the question in sequence, definition etc. are included in this.

(d) Problem Solving and Creative type Questions–In this type doing evaluation, proof it etc. are included.

But now essay type examinations are divided in this form :

(i) Long Answer Type Questions–The answer of this type of questions are in a broad way or long. No definite and fixed limit are to this. In this type of questions, the individual is asked to describe a thing, event, process for a particular person.

(ii) Short Answer Type Questions–Students give this type of questions in a definite limit. For example, what is Logarithm?

(iii) Very Short Answer Type Questions–The answer of this type of questions are in a word or in a sentence. For example, what is the sum of angles of a triangle? etc.

Merits of Essay Type Questions or Test

The followings are some major characteristics of essay type test :

1. Based on Broad Curriculum–In this the question can be asking the whole curriculum. These tests are helpful to decision to the knowledge and mental ability of the students.

2. Easy in construction and administration–These tests are easy in construction and also in administration. It is easy to formulate questions because the achievement of an individual is measured through the medium of one line question or two line question, as it takes very little time in formulating the questions, this is very useful in finding solution to immediate problems. Due to simplicity of the questions their administration is also easy.

3. Easy to check knowledge, interest and attitude–Teaching objectives of school subjects are divided into cognitive, affective and psychomotor domains. The students have to reply the essay type questions in detail, so their knowledge in the subject can be measured.

4. Evaluation of Personality–Essay type examinations are also very useful in evaluating the personality. With the help of the article we can measure the various aspects of personality as likes-dislikes, values etc.

5. Helpful in the development and preduction of writing power–With the help of essay type test we can measure the development and preduction of writing power of the student.

This also shows how well a person or student can compile his thoughts in a fixed time period. Thus, by these examinations a person's literary and writing ability can be measured.

6. Evaluation of Higher Mental Powers–By the help of essay type examination the high mental capabilities of an individual can be measured. This can be used successfully for problems where debating, review, point of view, analysis, synthesis etc. are required. This measures the thinking power, power of expression and intelligence.

7. Encouragement to Extensive Studies and Thinking–The answers to essay type questions have to be given in detail, so the students have to study extensively. Such tests encourage the students for extensive study and thinking.

8. Encouragement of Good and Detailed Study–This encourages detailed study with the help of various learning methods as the students have to give an answer of five to six pages to a question of two lines. Thus, he studies the subject-matter deeply. He prepares an outline for study, writes summary and tries to memorize important points.

9. Minimum Chance of Cheating–The answers of these tests are more lengthy. Due to the language style and compactness of the subject-matter the chances of cheating are minimised.

10. Practicability–We have been administering essay type tests from the beginning, so we have become habituated in the construction and administration of these tests and marking the answer books, and classifying the students on their basis. We do not face any difficulty in the administration. Thus, these tests are considered good from practical point of view.

Demerits of Essay Type Examination or Test

"..... bookish and mechanical, stereotyped and rigidly uniform and did not cater to the different aptitudes of the pupils."

—*Secondary Education Commission (1952-53)*

".....The whole systems of education is examination ridden. The frequency of examination and the manner of conducting them exercise an adverse effect upon the aims and methods of education. They suffer from a failure to define with any degree of exactness of the purpose."

—*The University Education Commission (1948-49)*

Some other demerits are as follows :

1. Obstructive in Physical and Mental Health–In these tests students study only during time of exam which are harmful to their physical and mental health. Too much emphasis an book-learning hinders physical growth from on early age. It does not take the account of day to day work of child.

2. Emphasis on Roting Power or Cramming–These questions develop the cramming power in the students. It gives emphasis on the memory power or cramming ability which do not develop the mental level and all round development. In other words it stimulates an unhealthy competitive spirit among children and encourage cramming of books and rote memorization rather than reflective thinking.

3. Time Consuming–Essay type examination is time consuming for students as well as for the teacher.

4. Lack of Reliability–The essay type questions do not give the same meaning. Different students understand them in different ways. It is but natural in such a case that there will be difference in their answers. And still further, the examiners too take their meaning differently. So it is almost necessary that difference in their measurement will creep in. It is evident that the results of such tests will not be subject to reliability. They are less reliable.

5. Lack of Validity–In essay type test there are no fixed objectives. There cannot be any validity about preduction because a student who gets

good marks in 10th class due to learning by heart may fail in successive classes. The teacher does not keep in mind the syllabus and level of the students while setting questions that is why there is absence of validity regarding syllabus. Thus, there is lack or absence of all types of validity.

6. Emphasis only on Expression–An important defect of the essay type examination system is that an outside examiner does not possess knowledge of the pupil's average work. Here expression effects the achievement of marks.

7. Emphasis on Writing and Speed–In essay type questions student has to write a lot. Taking an examination becomes an art, speed of writing is also an important factor.

8. Subjectivity–In essay type question, the subjective elements gain supremacy. The marking is influenced by the whims of the examiner. A number of researches have been conducted on it which have confirmed the subjectivity of essay type test or examination.

9. Half Representation of Curriculum–In essay type tests they do not represent the total or whole curriculum. Because in these tests 8-10 questions are included in the question paper which are the selected portion on the curriculum.

Hence about these demerit it is concluded that :

- It is an unclarity of objectives.
- Difficult in evaluation.
- Less Diagnosticity.
- Not Helpful in motivating the students.
- Effect of language and writing on the scores.
- It emphasises on memorisation of content matter rather than comprehension.
- Insufficient representation of syllabus.
- Actual evaluation of student's knowledge is doubtful.

Suggestion for Improvement in Essay Type Questions

The following suggestions are given for improvement in essay type examinations :

1. Increase the number of questions so as to cover the syllabus adequately.
2. The language of questions should be simple and their meaning should be unambiguous.
3. It should be used as projective technique to measure the higher mental abilities of students.
4. In this questions with their parts should be more clearly expressed and the distribution of marks for them should be made and shown.

5. The questions should be such on the basis of which the students can be classified into high, medium and low categories.
6. To improve its adıninistration. Administration of a test can be done perfectly only in a situation when the place is peaceful where execution of test is being done, the students have proper seating arrangement and the students are in normal physical and mental condition.
7. Further research may be made with trained examiners. Answer sheet can be examined by more than one examiner and average of marks given by them can be obtained.
8. There should be less choice of questions so that the actual knowledge of students could be tested.
9. The increase in number of questions and use of objective portions increase the reliability of the examination.
10. Advantages of well planned answer, instead of haphazard ones, may be emphasized and explained.

(ii) Objective Type Test

Objective type tests are new type tests. In these tests, short comings and limitations of essay type examinations were detalt by the educationists and psychologists. **Horace Mann** in 1845 A.D. developed the first objective test. **J.M. Rice** developed another objective test having only 50 words in 1897 A.D. **Afterward Starch** and **Thorndike** did commendable work in this direction.

In these tests, such questions are asked which have to be answered by making certain signs or writing a few words only. The answers to these questions are definite. The evaluators have not freedom in their evaluation. Because the answers and method of evaluation being definite, these tests are called objective tests.

According to **C.V. Good,** "Objective test is usually based on alternate response, multiple choice, matching or completion type questions and scored by means of a key of correct answers, any answer disagreeing with the key being regarded as wrong."

According to **Duglas and Holland,** "Objective tests are intended to obtain marks and grades that are relatively uninfluenced by the subjective factors."

Functions of Objective Type Examinations

- It provides help in educational and professional guidance and predicts future for them.
- It helps in classifying to the students.
- These tests defect the problems and weakness of the students and try to solve them

- It provides encouragement in the form of marks obtained and as this is objective it does not give a chance to anyone for cribbing.

Characteristics of Objective Type Test or Examinations

Following are the characteristics of objective type tests :

1. It takes lesser time to administer these test so the individual does not feel tired or bored.
2. There is no need to write in detail.
3. It is useful in selection, classification, promotion and other educational matter.
4. In these tests it is very easy and mechanical to award marks.
5. It is helpful to differentiate between the students.
6. It is based on educational objectives.
7. The number of questions in these tests is more so it is possible to take questions from the whole syllabus so the student's knowledge of the subject-matter can be measured in a proper manner.
8. It identifies quickness and of excellence of mind.
9. It is valid and more reliable.
10. It is a part from comprehensive knowledge.

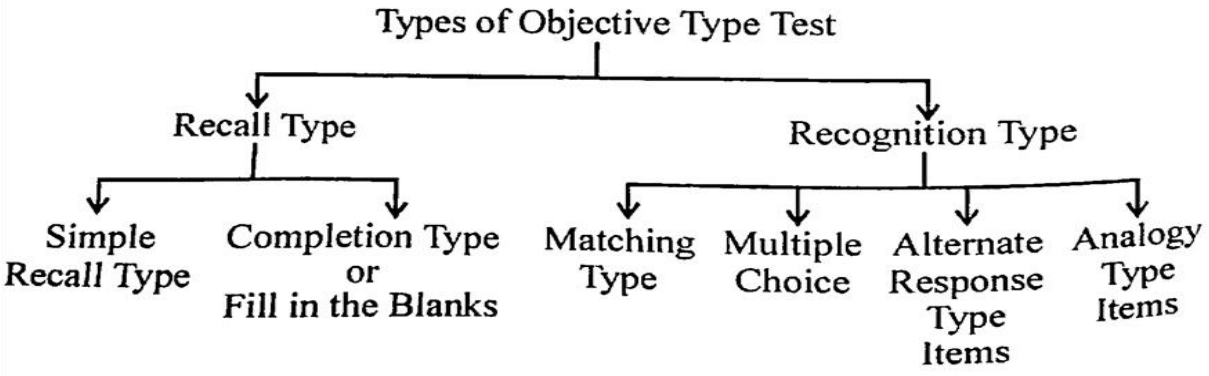
Kinds of Objective Type Test

Objective Type Test are classified into two categories :

1. Recall Type Questions
2. Recognition Type Questions

1. Recall Type Questions–In this type of questions there is repetition of previously learned facts thus they help in measuring the power of retention of the students. They are two types:

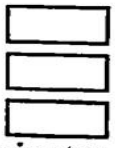
(a) Simple Recall Type Test or Questions–A simple recall type test is defined as one in which each item appears as a direct question. The nature of question is such that its answer is very specific and may be given precisely in a word or a number or a sentence. For examples :



(i) What will be the square root of 25?

(ii) What is the formula to calculate the area of circle?

(iii) What is the value of π?



This type of test measures familiarity and naturalness. It eliminates the guessing factor. One of the serious limitation of the simple recall item is that measures highly factual knowledge in bits or pieces.

(b) Completion Type Test or Fill in the Blanks–In this type of questions, the question is in the form of a sentence where one or more blanks are left for the student to fill in the answer. In this type there is no scope for guessing and the answer depends on understanding rather than memory. The main limitation of this test is that each item measures one or two bits of information. For examples:

(i) $\sqrt{3}$ is a ..... number.

(ii) The sum of all the interior angles in a triangle is ..... .

(iii) The ratio of circumference and diameter of a circle is ..... .

(iv) The point equidistant from the vertices of a triangle is ..... .

2. Recognition Type Item or Questions–In these questions, the question is to be recognized and answered accordingly. Although these help the examination in being objective, then too they are open to criticism because they promote guess work broadly. They are following type:

(i) Matching Type–A matching type item consists of two columns. Each statement in the first column is to be paired with a word in the second column. For example:

Match the Following

Column A	**Column B**
(a) Acute angle	(a) 90°
(b) Obtuse angle	(b) 180°
(c) Right angle	(c) 35°
(d) Straight angle	(d) more than 180°
	(e) more than 90° but less than 180°

Example II- Match the following :

Column A	**Column B**
(a) 4	(a) 64
(b) 8	(b) 81
(c) 5	(c) 144
(d) 9	(d) 16
(e) 12	(e) 25

(ii) Multiple Choice Questions–This type of questions include those questions of which several answers or solutions are presented and right answer has to be found out of these. For example :

(a) The average of 12, 14 & 16 is–

(i) 25 (ii) 14

(iii) 24 (iv) 36

(b) The value of $(a^2)^3$ is–

(i) a^5 (ii) a^{10}

(iii) a^6 (iv) None of these

(c) The value of tan (–150°) is–

(i) $-\sqrt{3}$ (ii) $-\frac{1}{\sqrt{3}}$

(iii) $\sqrt{3}$ (iv) $\frac{1}{\sqrt{3}}$

Multiple choice type items is most valuable and highly applicable type items. It is used to assess the judgment and discrimination power of the student. But in this type of items there is scope for guessing, cheating.

(iii) Alternate Type Items–These type of questions are asked in various ways as :

- True/False Questions
- Yes/No Questions
- Right/Wrong Questions

In this item only two possible responses are given. The pupil has to select one appropriate response from the given two alternate responses. This type of item makes the scoring, easy and objective. It is easy to construct and administer. But in this type of items there is much scope of guessing.

For example :

(a) $1 + \tan^2\theta = \sec^2\theta$ True/False

(b) Prime numbers include integers and fractions. True/False

(c) Pythagoras was an Indian mathematician. True/False

(iv) Analogy Type Items–It consists of two similar situations. One situation is incomplete. The pupil has to see the relationship in the first situation and on the basis of this relationship he has to complete the next situation. This test measures verbal reasoning. It may be recall as well as recognition type.

For example :

(a) 1 : 2 : : 50 : : .....

(b) $(a + b)^2 : (a^2 + b^2 + 2ab) : : (a - b)^2 :$ .....

(c) 4 : 16 : : 5 : .....

(d) sin θ : cosec θ : : ..... : cot (tan θ)

Selection Type Items–

(a) 49 : 81 : : ..... : 144

(i) 144
(ii) 64
(iii) 121
(iv) 100

(b) tan θ : cot θ : : ..... : sec θ

(i) sin θ
(ii) cot θ
(iii) cos θ
(iv) cosec θ

Merits of Objective Type Test

The major merits of objective type test are as below :

1. These tests are reliable. All the questions contained in them are clear and bear only one meaning, and their answers too are definite.
2. In these tests the student has to answer in a limited number of words so he cannot use ineffectual matter, chances of which are greater in essay type test or questions.
3. Due to reliable results, test is said to be scientific.
4. As these tests are prepared with a predetermined objective that is why they are more valid.
5. No need to go in detail, only brief and definite answers are required.
6. The questions asked in these tests are clear and have a single meaning, their answers are also definite and the evaluator has not any liberty to mark them. Whoever, the evaluator may be, the result of such tests is similar. It is evident that these tests are objective and therefore reliable.
7. In this test there is no effect of subjectivity of the evaluator.
8. Values to knowledge not efficiency and expertise in use of words.
9. Time consumption is not a problem hence test is absolutely economical.
10. Questions can be set from the whole syllabus, hence said to comprehensive.
11. Examiner can not be bluffed.
12. Free from complication, hence saves the students from misleading.

Demerits of Objective Type Tests

Despite having all the merits, these tests are not devoid of some shortcomings:

1. These tests examine the power of memory, the students have to hardly use the higher mental faculties such as logic and thinking.
2. Capabilities and talents of student can not be identified due to mechanical process of students mind.

3. It is difficult to find out the weaknesses and difficulties of the students through these tests.
4. Construction of objective tests are typical or difficult.
5. Lack of organization of thought in these tests.
6. The tests are prepared by those teachers who are already busy with their work, thus they have so much work that they cannot do justice to it.
7. Chances of cheating are more because answers are short and can be copied fast.
8. Their preparation requires a lot of labour, time and money.
9. Higher mental abilities cannot be measured by these tests.
10. It increases the burden on the teachers.
11. It does not provide knowledge about the student and his personality.
12. Possibility of correct answers only by guessing.
13. In these tests no opportunity for free expression of ideas.
14. Imagination and reasoning power also falls down due to short writing.
15. Extra-ordinary traits can not be exhibited.
16. In this tests language style and writing is completely ignored.
17. Standardization of objective test is a complex process.

Construction of objective based questions in behavioural Terms

Objective-1

Knowledge–To make the student that he can achieve the knowledge to scientific factors, steps, relations, concepts, techniques and theories.

Behaviour

1. They can recognise the steps, factors etc.
2. They can recall the steps, factors etc.
3. They can give the example of steps, factors etc.
4. They can give the definition of steps, factors and theories in his own words.
5. They can represent and explore the sign.
6. They can derive the simple results on the basis of facts, theories.

Objective-2

Understanding–To make the student that he can understand the scientific facts, steps, relations, concepts, formula, techniques, theories etc.

Behaviour

1. Student give examples.
2. Student found the errors.
3. Student found the relations.
4. Student classified them.
5. Student predicts the results.

Objective-3

Application–To prepare the student that he can use the scientific knowledge in new situations of daily life.

Behaviour

1. Student can achieve the known & unknown and can correlate in both of them.
2. Student can understand the use of data.
3. Student can understand the relation of data.
4. Student can assume the answers.
5. Student selects the proper method and theories.
6. Student solves the problem in a proper method.
7. Student can check the answer.

Short Answer Type Test

The tests in which such questions are asked, which can be answered briefly are called short answer type tests. The purpose of these tests is to test a large amount of knowledge, ability and understanding with in a short time. These questions are of two types:

(i) Short Answer Type Questions

(ii) Very Short Answer Type Questions

(i) **Short Answer Type Questions**–The questions which can be answered in comparatively less time and in less words than the essay type questions are called short answer type questions. For examples:

(a) What is the difference between measurement and evaluation?

(b) What are the steps of measurement?

(ii) **Very Short Answer Type Questions**–The questions which can be answered even more briefly and in shorter time than the short answer type questions are called very short answer type questions. For examples :

(a) What is the multiplicative inverse of (–5)?

(b) What do you mean by evaluation?

Merits of Short Answer Type Test

The merits of short-answer type test are :

1. The answers of these tests are generally definite, as such their measurement is objective and its is generally the case. It is evident that these tests are more reliable in comparison to essay type test.
2. It is more objective test.
3. Construction of these tests is easy.
4. Tests are generally used to measure change in the cognitive, affective and conative aspects. Though these tests can measure only cognitive aspects, yet they are so extensive that these are more valid.
5. They are more reliable than essay type test.
6. These questions give a vast sampling in lesser time.
7. These tests are administered just like essay type tests.
8. There is no possibility of guesswork.

Demerits of Short-Answer Type Tests

Some demerits of these tests are as follows :

1. These tests emphasise often on facts by which knowledge is chiefly tested. So, the students are inclined to learn by rote.
2. It takes more labour in reading the answer and awarding marks.
3. Inability to measure skills and attitude.
4. Inability to measure higher mental faculties such as logic.
5. The marking system is not objective as in objective type examination.

Suggestions for Improvement in Short Answer Type Tests

1. Their language should be simple and clear.
2. They have only one meaning.
3. The should instructions should be clear in the question.
4. The questions should be kept in proper ratio after a lot of thinking.
5. These question should be so framed as to have definite answers.

Difference between Essay Type and Objective Type Test

Essay Type Exam	Objective Type Exam
1. In this test the number of questions are less but their answers are long.	1. The number of questions are more but their answers are short.
2. Students give more time in thinking and writing.	2. In this students give more time in thinking and reading.
3. In this limited content are evaluated.	3. In this broad content are evaluated.
4. We can not standardize the norms of these test.	4. Standardized norms can fixed.
5. Construction of this test is easy but difficult to scoring marks.	5. Construction of this test is typical but easy in scoring.
6. Admnistration of this test is easy.	6. Administration method is difficult.
7. The validity and reliabity of these tests are of low level.	7. Reliability and validity are of high level.
8. In these tests teacher and student both are free.	8. In these tests teacher and students both are not free.
9. It is based on a small sample.	9. It is based on large sample.
10. These tests are more useful to achievement tests, selection and classification.	10. These tests are more useful to performance test, diagnosis, intelligence and aptitude test etc.

Suggestion or Recommendations of Various Commissions Regarding Improvement in Evaluation

(A) University Grant Commission (U.G.C.) (1948-49)

Commission gaves many suggestions to improve the exam :

1. University and education ministry should develop scientific method of the educational measurement and evaluation.
2. One or two experienced persons should be appointed.
3. Arrange the committee of examiners in each university.
4. Development of the appropriate psychological and performance tests for higher secondary examination.

(B) Secondary Education Commission (1952-53)

1. Decrease the number of external examinations and essay type questions.
2. School should be keep the whole progress report of the students.
3. Internal test should be important for progress of students.
4. Only one last external exam should be arranged in secondary curriculum.
5. Objective and short answer type questions should be arranged in place of essay type examinations.

ACHIEVEMENT TEST

Achievement test is tool for teachers for evaluation of students in school situation. With the help of achievement test we can measure the amount of success of an individual in specific field. In school environment it is used as an instrument to measure success of an individual in particular subject or group of subjects. It gives the knowledge about what an individual acquire by testing his abilities.

Definitions

- According to Waters, "Achievement test act as useful aids in diagnosing the student's specific learning needs for identifying his relative strengths and weaknesses".
- According to Super, "An achievement test or proficiency test is used to ascertain what and how much has been learnt or how well a task can performed".
- According to Free Man, "Achievement test is a test designed to measure knowledge, understanding and skills in a specified subject or a group of subjects".
- According to N.M. Downie, "Any test that measures the attainments or accomplishments of an individual after period of training or learning is called achievement test. It helps to permute the student to next class."

Uses of Achievement Test

1. For Administrator's Use:

1. Test helps to evaluate the extent to which the objectives of education are being achieved.
2. To evaluate, revise and improve the curriculum in the light of results obtained.
3. Tests help to classify school objectives.
4. Tests discover the type of learning experiences that will achieve these objectives with the best possible results.
5. To select talented pupils for special classes and courses.
6. To select students for the award of special merits or scholarships.
7. To discover backward children who need help and to plan for remedial instruction for such students.
8. To decide for proper classification of students.
9. To group students in a class, so that they will be put in such a way that individual differences are as little as possible.
10. To determine the general level of achievement of a class and thus to judge the teaching efficiency of the teacher. The level of achievement of a class may be

judged on the basis of the achievement of the class in the beginning and at the end of the school year.

11. To determine the efficiency of one school with the others.
12. To help the parents in recognizing the strengths and weaknesses of their children so that they direct their energies on suitable goals only and don't put heavy demands on them.
13. To get a better understanding of the needs and abilities of pupils.

2. The Teacher's Use:

1. The teacher will come to know the general range of abilities of students in the class.
2. In the light of the above, he will select appropriate materials of instruction, so that all individuals benefit from instruction to the maximum.
3. The teacher will determine and diagnose the weakness of the students in various subjects.
4. The teacher will spot brilliant and backward children.
5. He will determine the progress of the group in a particular subject over a period of time.

9.2 Construction of Achievement Test

If the test is to serve as an efficient and effective tool of evaluation, its preparation should be based on a number of careful considerations. The preparation of a good test is a systematic process having well-defined stages. The important steps envisaged in the preparation of a good teacher – made test are.

- Planning the test
- Preparation of a design
- Designing the test items
- Reviewing and editing
- Arranging the items
- Providing directions
- Preparing the scoring key and marking scheme
- Administering the test and scoring
- Evaluating the test

9.2.1 Planning the Test

"Test planning encompasses all of the varied operations that go into producing the test; but it must also involve careful attention to test item difficulty, to type of test items, to directions to the examiner". (Lindquist, E.F.). We shall cover most of these considerations under the heading; preparation of design for the test or blueprint of the test.

9.2.2 Preparation of Design

Designing is the first and most important step in the test construction. It is at this stage that we plan to build in the test the important qualities: validity, reliability, objectivity and practicability. In order to accomplish this, the test constructor has to take a number of decisions regarding selection of the objective, the selection of the content, form of questions, the difficulty level of tests items and the weightages to be allotted to the objectives, to the content and the form of questions. The set of those decisions will be called the *design of the test* from an analogy with the work of an architect. Important decisions have to be taken concerning the following.

i) Identification of the objectives and allotting weightage to the objectives

The most important step while planning a test, or any evaluation tool or technique for that matter, is the identification of the instructional objectives

and stating them in terms of specific observable behaviour. After the objectives are identified and stated, the test maker has to decide their relative weights in the test. The fundamental principle to be observed here is that the test should reflect the actual emphasis being given to various mental processes during instruction. Because of the intangibility of instructional process, there cannot be any fixed formula for assigning weights to various objectives. But these weights will be by and large a function of time, effort and resources spent on their acquisition as also for the importance for the society and the learning of the subject in terms of retention and transfer value. The simplest basis for assigning the weights could be to weigh them in terms of time devoted for their achievement. Weightages could, of course, be given in numerical terms, distributing 100 points over the objectives, giving the greatest number to those that are to receive the greatest emphasis.

For the sake of illustration, given below (table 9.3) are the objectives identified for teaching some units in mathematics and the weightage allotted to them.

Table-9.3: Showing the Weightage Allotted to the Objective

S.No.	*Objectives*	*Marks Allotted*	*Percentage*
1	Knowledge	6	24
2	Understanding	7	28
3	Application	10	40
4	Skill	2	8
	Total	25	100

ii) Selection of the content and allotting weightage to the content

Content being the means through which objectives are attained, it becomes very necessary to decide the weights to be given to its different parts. As the whole syllabus cannot be covered through any single test, a convenient number of units can be selected for testing. When this is done a decision about the weights to be given to these units has to be taken so as to represent the actual emphasis on them in instruction. In assigning relative weights to units a number of factors will have to be taken into account. How important is the unit in the total learning experience? How much time was devoted to it during instruction? Although there are a number of such considerations, the easiest method to decide weightages may be to base them on the time required to teach various units.

For the sake of illustration, given below is the table 9.4 indicating the weightages given to three units namely Polynomials, Functions and Quadrilaterals.

Table-9.4: Weightages given to Polynomials, Functions and Quadrilaterals

S.No.	*Unit*	*Marks Given*	*Percentage*
1.	Polynomials	10	40
2.	Functions	8	32
3.	Quadrilaterals (Construction)	7	28
Total		25	100

iii) Selection of the form of questions and giving weightage to the questions

The test-maker has to decide about the form of questions to be used, the number of questions to be chosen and the relative weightage to be given to each form. Perhaps a judicious combinations of the different forms will have to be used in achievement tests. However, among the objective type questions, multiple choice may be given more weightage.

In the question paper which is being taken for illustration, the weightage to different forms of questions could be as follows. (table.9.5)

Table-9.5: Weightages given to Different Forms of Questions

S.No.	*Form*	*Marks Given*	*Percentage*
1.	Essay (E)	6	24
2.	Short answer (S.A)	9	36
3.	Objective type (O)	10	40
Total		25	100

iv) Distribution of difficulty level

A decision also has to be taken concerning the distribution of difficulty level. The distribution of difficulty level in a test will depend upon the purpose of the test as also on the group of students for whom it is designed. To get optimum discrimination through a test, most of its questions should be of average difficulty level. A few easy questions, to motivate the below average students and a few difficult ones to challenge the gifted should find a place in the question paper If achievement can be assumed to be normally distributed, some weightages in terms of percentages can be suggested for easy, average and difficult questions as shown in table.9.6.

Table-9.6: Distribution of Difficulty Level for Questions

S.No.	*Difficulty Level*	*Percentage*
1.	Difficult Questions	15
2.	Average Questions	70
3.	Easy Questions	15
	Total	100

v) Preparation of Blueprint: (Table of specifications)

Preparation of the blueprint refers to the final stage of the planning of a test. The blue- print is a three dimensional chart showing the weightage given to the objectives, content and the form of questions in terms of marks. It is also called a table of specifications as it relates outcome to the content and indicates the relative weight given to each of the various areas. The units or the content spread along the vertical axis while the objectives are listed on horizontal axis. Each column is further subdivided into columns that indicate the forms of questions. Thus we get a number of cells, each cell having three dimensions, the objective, the content and the form of question. What is required on the part of the test maker is to fit in all the questions in different cells in such a way that the blueprint reflects the decisions of the design(er). When this is done, all rows and columns are balanced, and the blueprint is ready. It is illustrated in table.9.7

9.2.3 Designing the Test Items

When the blueprint is ready, the next step is to prepare or select the items. Write the test items according to the table of specifications. Take each cell of the blueprint and draft an item taking care of the various dimensions; the objective, the content and the form as laid down in the blueprint. Some other decisions taken while designing the test, particularly the one concerning the distribution of difficulty level has to be implemented at this stage itself. It is also desirable to prepare more items than the requirement of the cells of the blueprint since defects are likely to become apparent in some items during the later review. The additional items will make it easier to maintain the distribution of items reflected in the table of specifications.

9.2.4 Reviewing and Editing

The pool of items for a particular test after being set aside for a time can be reviewed with the help of experts. A more careful evaluation of the items can be made by considering them in the light of each of the following questions.

Table-9.7: Blueprint

Sl.No.	Objectives / Content	Knowledge			Understanding			Application			Skill			Total
		E	SA	O	E	SA	O	E	SA	O	E	SA	O	
1.	Polynomial			1(2)		3(1)	1(1)		3(1)	1(1)				10
2.	Functions			1(2)		3(1)	1(1)			1(2)				8
3.	Quadrilateral			1(1)				4			2			7
	Total	5			8			10			2			25

Note: The number inside the bracket indicates the number of questions and the number outside the bracket indicates the marks allotted to each question.

Uses of Blueprint

The preparation of the blueprint serves the following purposes. The blueprint

- helps to improve the content validity of the test.
- defines as clearly as possible the scope and emphasis of the test.
- relates objectives to the content.
- gives greater assurance that the test will measure learning outcomes and course content in a balanced manner.
- lays before the tester a complete picture of the test he is going to prepare.

- Does each item measure an important learning outcome included in the table of specifications?
- Is each item appropriate for the particular learning outcome to be measured?
- Does each item present a clearly formulated task?
- Is the item stated in simple, clear language?
- Is the item free form extraneous clues?
- Is the difficulty of the item appropriate for the students to be tested?
- Is each item independent and are the items, as a group, free from overlapping?
- Does each item fit into one of the cells of the blueprint?

9.2.5 Arranging the Items

When the final selection of the items has been completed and they are ready to be assembled into a test, a decision must be made concerning the best item arrangement. The following suggestions provide guidelines for this purpose.

- The items should be arranged so that all items of the same type are grouped together.
- The items should be arranged in the order of increasing difficulty.
- It may be desirable to group together items which measure the same learning outcomes or the same subject matter content.

9.2.6 Providing Directions

Directions constitute an inseparable part of a test. The directions for an achievement test should be simple and concise and yet contain information concerning each of the following:

1. purpose of the test
2. time allowed to complete the test
3. how to record the answers
4. whether to guess when in doubt about the answer
5. marks allotted for each question as also for each section of the test.

9.2.7 Preparation of Scoring Key and Marking Scheme

When the test has been assembled and ready to administer, it becomes necessary to prepare some other important accessories in the form of a

scoring key for objective type questions and marking scheme for supply-type questions, such as short answer and essay type questions.

Scoring key

Scoring key refers to the prepared list of answers to a given set of objective questions. The examiner compares the answers given by the students with those in the scoring key and thus arrives at the marks to be awarded to the students.

Correction for Guessing

The test constructor while preparing the scoring key may feel like reducing to zero the gain in the score expected to result from guess work. This is popularly called correction for guessing.

The generalized formula for correcting for guessing is.

$$S = R - \frac{kw}{n-k}$$

where

S = Score,

w = Number of wrong responses,

R = Number of right responses.

n = Number of suggested responses for a single item.

k = Number of responses to be selected or marked for each item.

In the case of True or False items,

$S = R - W$ (since $n = 2$, $k = 1$)

In the case of multiple choice questions where only one correct answer is expected, the generalized formula reduces to $S = R - \frac{W}{n-1}$

Marking Scheme

In the preparation of marking scheme in mathematics it will be desirable not only to analyse the solution into important stages and to distribute marks over them but each stage may be looked upon from the point of view of the method involved as also from expected accuracy. The marks for each stage, therefore, may be divided into two components: marks for the method and those for accuracy. A marking scheme is essential because it indicates:

i) The number of steps or learning points expected in the answer

ii) The outline of each point or step expected in the answer

iii) The weightage to each point or each step

iv) The level of accuracy expected of each step

9.2.8 Administering and Scoring the Test

At this step, it is important to make sure that all students know exactly what is expected of them and to provide them with the most favourable conditions for taking the test. After the administration of the test, the scoring can be done with the help of the scoring key and marking scheme. An illustration for the scoring key and marking schemes are given in tables 9.8 and 9.9

Table-9.8: Illustration for the Format of Scoring Key

Q.No	1	2	3	4	5	6	7	8	9	10
Expected Right Answer	A	B	B	C	A	25.Sq.cm	$(x+2)(x+1)$	—	—	—
Marks Allotted	1	1	1	1	1	1	1			

Table-9.9: Illustration for the Format of Marking Scheme

Q.No.	*Value Point Expected Answer*	*Marks allotted for each step*	*Total marks*
1	For writing the definition of the set correctly	1 mark	3
	For writing at least one example for the set	1 mark	
	For writing the Set in the:		
	(i) roaster form	½ mark	
	(ii) set builder form	½ mark	

9.2.9 Evaluating the Test

Evaluating the test helps the teacher to ascertain the following:

- Was the test very easy or very difficult?
- Was the test too long or too short?

- Were the directions clear and specific?
- Was the test practicable and feasible?
- Did the items measure the intended objectives?
- Did the difficulty level of the questions match the level of the students?
- Were the items clear and unambiguous?
- Were the distracters effective?
- Do the items discriminate among the different levels of achievers?
- What are the misconceptions formed by the student?
- How effective were the learning experiences provided during the instruction?

Evaluation can be done at two levels:

i) Question-wise analysis

ii) Item analysis

Question-Wise Analysis

Question-wise analysis is done by analysing each question according to objective, specification, content, question type, estimated difficulty level and time required. Format for the question-wise analysis is given in table-9.10. This helps the teacher in assessing the effectiveness of the test item with reference to the objectives and other requirements in the blueprint. This also gives an insight into the difficulties encountered by students in taking the test.

Table-9.10: Format for Question-wise Analysis

Q.No	*Objective*	*Specification*	*Form of Question*	*Unit*	*Marks allotted*	*Time in Minutes*	*Difficulty Level*
1.	Knowledge	Recalls	Objective (MC)	Set Theory	1	1	E*
2.	Understanding	Identies	Objective (MC)	Simultaneous Equation	1	1	A*
3.	Application	Selects	Objective	Set Theory	1	1	D*

***E=Easy; *A=Average; *D= Difficulty**

Item Analysis

Item analysis is the process by which the test maker evaluates the effectiveness of the test item in terms of (i) the difficulty level of the test items (ii) discriminating power of test items and (iii) the effectiveness of the distracters. For item analysis the teacher arranges the test papers in the

ascending order of marks and analysis is done for 27% of the students on the high and low end of the scales and making sure that there are an equal number of students in both the groups. Students' responses to each item is analysed for the students in upper and lower groups. Other than the purposes mentioned above, item analysis has several other benefits too.

- It provides useful information for class discussion of the test.
- It provides data for helping the students to improve their learning.
- It provides insight and skill which lead to the preparation of better tests on future occasions

ITEM ANALYSIS:

The test is evaluated with the help of scoring key. It is assumed that the test was very easy for the class, if the scores seem extremely high. Similarly, the test was probably very difficult if the scores are very low. It is advisable to review the question paper in following steps:

1. Question wise analysis -before the test
2. Critical evaluation of the test -before the test
3. Item analysis -after the test
4. Preparation for Final Draft

Question wise analysis helps to find out the weakness and strength of the test; to relate the question paper with the blueprint; and to inquire the content validity of the test. Each question is analyzed according to the objective specification, topic, question form and type, estimated difficulty level, time needed, and marks allotted.

Critical evaluation of the test helps to bring out any replication, spelling errors, ambiguities in the paper. A qualitative and quantitative assessment of the test must be done.

Item analysis enables the teacher to determine the difficulty value of each item; the discriminating power of each item; and the effectiveness of distracters in the given item.

Preparation for Final Draft: Test constructor analyses the responses of students for each test item. The whole data is gone through a statistical procedure. For the final draft, appropriate items are selected and inappropriate items are removed. A systematic arrangement of questions is done in the final draft, so that a clear identity of test is appeared. A final look of test is given by mentioning instructions to attempt the test, time allotment and responder's particular filling spaces or boxes at the top.

COMPREHENSIVE AND CONTINUOUS EVALUATION IN MATHEMATICS

Assessment of performance is an integral part of any process of learning and teaching. As a part of sound educational strategy, examinations should be employed to bring about qualitative improvement in education.

—The NPE, 1986

There is an urgent need to make evaluation an integral part of the whole teaching-learning process. The traditional examinations take cognizance of the scholastic area only, the attention and efforts of both teachers and children goes primarily to this area. Therefore, present school programmes also neglect many vital areas of a child's personality by starving them from the required inputs. Infect, if teaching-learning process is continuous, the examination or evaluation should also be continuous. The Comprehensive and Continuous Evaluation (CCE) provides accomodation for individual differences. It aims at fostering individual ability of children and helps them to realize their potentialities. The CCE also aims at making up the deficiency by laying adequate emphasis on the development of non-scholastic areas. Thus, it helps to develop all aspects of child's growth to his/her optional potential. Therefore, the comprehenive and continuous evaluation presents a combination of external and internal evaluation. In a system of comprehensive and continuous evaluation both scholastic and non-scholastic aspects of education are to be kept in view.

Comprehensive Evaluation

One of the main defects of the present examination system is that its coverage is very limited to academic aspects. The non-academic aspects. The non-academic aspects are not attempted for the allround development of the child's personality. All the efforts for instruction are focused on this limited area. So, the education remains confined to the cognitive domain alone and even there it covers very little beyond memorisation. Hence, the objective of the harmonious development of child's personality, remains unfulilled.

The several areas have been added to the curriculum, which facilitate the development of children in non-academic areas. The several areas are : work experience course, health education, physical education and other specially planned co-curicular activities. Child's growth in non-cognitive areas should be evaluated and guided

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continuously and it may not be the part of a formal examination. In this context the concept of comprehensive evaluation covering different aspect of a child's growth had conceived.

Continuous Evaluation

It is likely possible to obtain valuable data about the strength and weaknesses of the children through continuous evaluation. Continuous evaluation helps in providing remedies and enriched instruction with a view to realising the objectives of education in the optimum growth and development of various aspects of child's personality. The feedback helps teachers in improving the level of achievement and proficiency among children.

More over, the comprehensive and continuous evaluation will be a part of a teacher's duty. The scheme of comprehensive and continuous evaluation can work only, when there is healthy teacher-taught relationship. Hence, comprehensive and continuous evaluation is not an end in itself. It provides opportunities for teachers to make suitable changes in their efforts. Continuous feedback also provides direction to children and parents in their efforts. The **National Policy on Education** (1986), has also accepted that assessment of performance is an integral part of any process of learning and teaching. As apart of sound educational strategy; examinations should be exployed to bring about qualitative improvement in education. Evaluaion will be streamlined and the predominance of external examinations will be reduced in education.

Objectives of Comprehensive and Continuous Evaluation

The main objectives of CCE are as follows :

- To foster individual abilities of the children.
- To help the children to realize their potentialities and capacities.
- To enable teacher to evaluate those attitudes. Abilities and skills which are impossible to evaluate through traditional examinations.
- To help teacher in improving the level of achievement.
- To emphasise the development of non-scholastic areas.
- The elimination of excessive element of chance and subjectivity.
- The elimiation on emphasis of memorization.
- To help the children to have periodical feedback to judge their achievements.
- To help teachers to constinuously update their judgement in respect to the progress of the children in the cognitive, affective and psychomotor domains.
- To provide remedial and enriched instructions.
- To develop all the aspects of child's growth to his/her optional potential.
- To motivate both the teachers and children to improve the teaching-learning process.
- To evaluate comprehensively the more important abilities like; affection, certain values, attitudes, interests, habits, etc.
- To obtain valuable data about the strengths and weaknesses of the children regularly.

- To provide opportunities for teachers to make suitable changes in their efforts.
- The continuous feedback also provides direction to children and parents in their efforts.

Scope/Areas of Comprehensive and Continuous Evaluation

- The scheme of comprehensive and continuous evaluation covers the following aspects and areas of personality of a child.
- Academic achievement of the child.
- Personal and social qualities; such as : regularity, responsibility, punctuality, habit of cleanliness, Co-operation, initiative, sense of social service etc.
- Desirable attitudes; such as: socialism, secularism, democracy, attitude towards teachers, school, studies national integration etc.
- Interests; such as : Cultural, artistic, literary, scientific etc.
- Proficiency in co-curricular activities; such as: games, sports, scouting-guiding. First aid, Red-cross etc.
- Health status (heigth, weight, chest expansion, cleanliness, freedom from disease etc.,) which a teacher can observe and record.

Components of CCE

Various tools and techniques such as observation of pupil behaviour in different situations, checklists, inventories, rating scales, records of interests, initiative, and creativeity etc., may be used for evaluation in non-academic areas. Cumulative records will show growth charts of the children. The evaluation attempted on these lines will also be more revealing and are of better use for childern, parents and teachers. Various components of comprehensive and continuous evaluaton may be as follows :

- **Quizes**—Scheduled or unscheduled.
- Assignments—Class Assignments & home assignments.
- Written and oral tests—objective, short-answer, essay type, problem-solving.
- Practical/Field/Laboratory/Project work.
- Term papers/Dissertation/Thesis/Survey reports.
- Tutorial group activities/Seminar/group discussion.
- Weightage to attendance—distribution of marks on the basis of presence/ absence.

Advantages of Comprehensive and Continuous Evaluation

- CCE based on the assumption that the abilities and capacities of childern are evaluated continuously at every stage.
- It develops the habit of library consultation.
- It develops the habit of self study as well as regular study.
- It helps to develop confidence in the child.
- It provides the opportunity for mutual consulation.
- It also helps to develop the habit of advanced preparation of the lesson/ topic.
- Seminars, group discussion, tutarials etc. lead to a matter of interaction between the teacher and pupil and also between students and student.

- It helps to remove the weakenses or deficiencies in teaching-learning process.
- With the remedial programms and projects the foundation of concrete and constructive teaching-learning may be laid down.
- CCE incorporates both cognitive and non-cognitive aspect of the child's personality.
- It eliminates the excessive element of chance and subjectivity.
- It provides periodical feedback to the child and teacher both.
- It provides valuable data about the strengths and weaknesses of the children regularly.
- It provides opportunities for teachers and children to make suitable change in their efforts.
- It provides opportunities for remedial teaching and enriched instructions.
- It develops all the aspects of child's growth for his/her optimal potential.
- It also helps teachers to continuously and comprehensively update their judgements in respect of the progress of the children in all the three domains of behaviour.
- CCE motivates both the teachers and children to improve teaching and learning process.
- CCE fosters individual abilities of the children.

Limitations of Comprehensive and Continuous Evaluation

- The CCE may work only when there is happy and healthy teacher-taught relationship.
- Some teachers and examiners threaten their students and make then insecure as well as mutinous.
- It is not possible for a teacher to do CCE without knowing his students and their background.
- Favoritism, biases and prejudices of teacher can make the CCE subjective rather than background.
- The CCE may not be successful in a crowded class-room.
- It is more laborious and time consuming.
- This can be done by a true and devoted teacher.
- The observations and judgements of teacher may be free from favourites, prejudices and partiality. *i.e.* Teachers may fail to maintain objectivity in their role.
- It teacher fail to demonstrate integrity, sense of impartiality and resistance to pressure, CCE may become a mockery.

Present Examination System

The **Radha Krishanan Commission** (1948) was of the opinion that the prevailing system of the worst features of Indian education and stated : *"If they were to suggest one single reform in university education, it would be that of examination."* The influence, both positive and negative, of the examination system on education in general and secondary education in particular has been discussed in the reports of several committees and commissions. Reviewing **the defects of examination, at the secondary stages, the secondary education commission noted.**

"The examinations today dictate curriculum instead of following it, prevent any experimentation, hamper the proper treatment of subjects and sound methods of teaching, faster a dull uniformity rather than ***originality,*** *encourage the average pupil to* ***concentrate*** *too rigidly upon too narrow field and thus help him to develop wrong values in education."*

Defects of Present Examinations System

All educationist and administrators have bitterly denounced the present system of examination. The wholly unsatisfactory characteristics of our method of testing students has been underlined by may authorities. The painful effect of all examinations is that we have made a nation memorizers and crammers. So, education becomes examination **Centred** instead of student-centred. The major defects in the present system of examination can be outlined as follows :

(i) Subjectivity—Subjective attitude of examiners influences the marks obtained by an individual and leads to startling variability in marking. Therefore, a deserving student may fail and an undeserving one may pass. Thus, the results are declared in terms of raw marks which suffer from a number of inadequancies.

(ii) Element of Chance—The fate of large number of students is decided through examinations. It is often found that some children prepare a few selected questions and totally depend on chance. If **by-chance,** these selected questions appear in the examination, they get through the examination excellent credits and such children are considered superior to those who sincerely study the whole year, but unable to do well in the examinations due to many reasons.

(iii) Ignoring Quality of Character—The present examinations do not provide measure to test the originality, initiative, truthfulness, honesty, and sociability of the children. Without these qualities the true objectives of education cannot be achieved.

(iv) Lowering of Moral Standards—Existing examination system adversely affects the moral standards of the children. The high ambition to get through the examination in spite of basic efficiency prompts the children to resort to immoral practices, unsocial behaviour, indiscipline and use of unfair means.

(v) Glorification of Memory—The present examinations test only artificial and superficial layers of facts. They give the children neither breadth of vision nor foster historical thinking in their minds and test the intelligently learning of facts that are related to daily-life in the same manner.

(vi) Heavy Mental Stress and Strain—Present examination system exercises a heavy mental stress and strain on the children. The children get panic and fear struck. This fear rot their originality. Clear thinking and imaginativeness. **Kothari Commission** (1964-66) has also remarked that present examinations cause unhealthy rivalries among the children. The fear of failure some times leads to extreme measures like suicide.

(vii) Present examination system focuses on one aspect of children's achievement *i.e.* academic aspect and ignores the non-cognitive aspects which are a vital component of human personality.

(viii) It covers a small fraction of the course content that the pupils strive to learn over a period of one year.

(ix) It resorts to the use of written tests.

(x) It does not provide opportunities for the application of multiple techniques of evaluation like: oral, observations, assignments, projects, supervised study etc.

(xi) It also suffers from an unhuman rigidity which allows little flexibility of any kind.

(xii) The teachers who teach, have no opportunity to asses their own teaching.

(xiii) It fully based an external assessment of the child.

(xiv) It does not test the creative and construction ability of the children.

Advantages of Examinations

Therefore, on the basis of the defects and short commings of the present examination system, many educationists are so much disgusted that they have advocated it complete abolition but we can not abolish it altogether. Examination in one form or the other is essential. Teachers, presents, students, or one claim that the whole society feels it utmost need. **There are incidental advantages of examinations :**

- Examination does motivational work.
- It serves as an incentive to pupils in their work.
- It discloses defects in teaching and learning.
- It provides training in organising and using knowledge.
- It is an important function of our educational system.
- It provides us with a measuring instrument, measurement and evaluation that are essential part of education.
- The whole society feels its needs and importance.

Suggestions for Improving the Existing Examination System

On the basis of above shortcomings of existing examination system, it is clear that present system of examination in our country is not capable of measuring the real abilities of children and needs comprehensive reforms. The major suggestions are as follows :

(A) Improving External Examinations

- Content coverage should be in accurate form and overall options should be eliminated as they lead to selective study.
- Questions should be made specific, objective and exacts.
- The content should be consisted of a representative sample of every aspect of the domain in which achievement is to be assessed.
- The content and the organisation should make it possible for a consistent evaluation of the achievement of each candidate.
- To eliminate subjectivity in marking scoring procedures should be improved.
- Teachers should adopt the new techniques of evaluation.
- Model questions should always be prepared for each exam.

(B) Improving Internal Examinations

- The teachers should be trained for objective and impartial evaluation.
- Continuous evaluation throughout the year may be supplemented through an annual examination, that will minimize the chance factor in examinations.

(C) Improving Written Examinations

- The proportion of the essay type, short-answer type and objective type questions in any paper may vary from subjects to subject.
- It is desirable to introduce the short-answer type questions increasingly.
- Elements of subjectivity in examinations should be reduced through objectivity.
- All the major areas of content should be covered in the question papers.
- The chances of copying, using unfair and illegal means should be minimised.
- The method of spot evaluations should be adopted *i.e.* checking answer book/copies at one central place.
- In final examination certificate, the results should be in two columns : Internal assessment and external assessment.

(D) Introducing Continuous and Comprehensive Evaluation

- It may be helpful for a better insight into the personality of the children.
- Over all continuous and comprehensive evaluation of a child should be done on the basis of his complete sessional work/assignments.
- Sessional assessment which provides a measures for number of important abilities *viz;* derives and capacities for hard work, motivation, imagination institution and speculation leadership and team work etc.
- Sessional work/assignment may include writing of essays, project reports, home assignments, Group discussions, seminars field work, laboratory work, tutorial activities, survey reports, etc.
- Grading system should be introduced in place of marking.
- A cummulative record of profit should be maintained in every school for the proper assessment of physical, mental, moral, social and emotional aspects of the children.
- Question Bank should be developed in all the subjects.
- The system of the called 'final examination' should be replaced by assessments over all well distributed intervals.

Recommendations of Kothari Commission (1964-66)

The Education Commission (1964-66), critically examined the present examination system and recommended that :

- Evaluation should be considered as an integral process and part of teaching-learning process.
- The new approach of evaluation should be attempt to improve the written examination.
- For improving the nature of questions, there should be adoption of scientific procedures of scoring and processing of results.

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- The internal assessment should be shown seperately form the external examination marks.
- Written examinations should be improved.
- External examination should be improved by raising the technical competence of paper-setters, objective-oriented question papers etc.
- The internal assessment of school should be comprehensive and evaluate all aspects of student growth including those not measured by external examinations.

Keeping all these factors in view, it will be appropriate and very timely for the country to pay need to the Recommendation of the Programme of Action, 1992 that *"the predominance of the external examinations should be reduced."* Hence, evaluation must facilitate all-round development of children. It will be, therefore desirable to have report-based system of child's evaluation.

Examination Reforms

Examination is necessary exercise in the process of education. It is a measure of educational achievement as an indication of the level of fulfillment of the objectives of education as a guide to employment examinations are indispensable. Examinations have been referred to as the central nervous system of education. This is because examination has a determining effect on carriers and hence attract the emotional environment of the students. Examination Reform, has somehow come to occupy the most central position in the public eye. The urgency of the need for reforming examinations has also evoked the active interest of the Govt. of India with a view to getting some tangible results in the quickies possible time in this crucial area of education. It focuses attention on the two following major goals :

- Improving examinations so as to make them reliable and valid measures of pupil's growth.
- Improving examinations so as to make them powerful instruments of improving teaching and learning.

The above major goals of examination reform are relevant for both external examinations and evaluation in schools. In functional terms these goals would mean continuous and comprehensive evaluation incorporating both the scholastic and Non-scholastic aspects of pupil's growth.

Areas of Examination Reforms

There are many areas of examination reform, Some of these are as follows :

[I] Reforms in Written Examinations

- Improvement of questions and question papers.
- Improvement of scoring procedures

Through the measures like :

(a) Developing detailed marking scheme by the paper setters
(b) Introduction of innovations like centralized evaluation and randomizations of scripts for purposes of evaluation.
(c) Ensuring uniformity of assessment standards; evaluation of a sample of scripts by a member of examiners followed by a discussion among them.

Improvement in the Mechanics of Conducting Examination Through Steps Like :

(a) Deciding cut scores for awarding grades.
(b) Scaling of results for ensuring uniformity of standards.
(c) Devising improved methods of collecting, compiling, tabulating an analysing data for effective feed back.
(d) Introducing grades in place of marks.
(e) Conducting functional research into the examination problems.
(f) Provision of fictitious roll members to answer sheets before sending ther for evaluation.
(g) Including various type of questions like objective type, short-answer, very short-answer fixed response type, etc.

[II] Reforms in Practical Examinations

(a) Evaluating the proficiency in a wider variety of skills.
(b) Evaluating both the process and product of performance.
(c) Developing a detailed marking scheme, *i.e.* minimising inter-examiner and intra-examiner variability in scoring.

[III] Reforms in Oral Examinations.

- Oral examinations in language subjects.

(a) identification of linguistic abilities of oral expression
(b) comprehensive covrage of various linguistic abilities of oral expression
(c) improvement of objectivity in the assessment of these abilities.

- Oral examination in subjects other than languages.

(a) general questions of create a report
(b) specific to judge the level of accuracy of the acquired knowledge
(c) cross-questionning to judge the depth of knowledge of the pupils.

[IV] Steps towards making evaluation continuous and comprehensive to cover both the scholastic and non-scholastic areas of pupils growth.

- **Academic Operations**

(a) identification of different aspects of personality
(b) development of evaluative criteria for the assessment of these areas.
(c) preparing tools for evaluating various identified aspects
(d) developing procedures for organising programmes and activities
(e) developing guidelines for collecting, recording and interpreting the evidences
(f) improving the proficiency of the teacher *i.e.* feedback.

- **Administrative Operations**

(a) providing a separate certificate of internal assessment along with external examinations
(b) developing and enforcing effective built in methods
(c) ensuring the equivalence of school standards.

Hence, models of examination reform developed in India by the NCERT have admirably succeeded, where appropriate inputs have been invested into the programme in adequate measure and at appropriate manner.

9.3 Statistical Measures

Item analysis and item-wise analysis carried out after administering the tests, help the teacher in assessing the effectiveness of the test items and in assessing the overall performance of the students. However, the test scores should be subjected to statistical treatment for further interpretation and drawing valid inferences.

The statistical analysis of the scores helps the teacher to describe or summarise the test score which facilitates objective comparison of student performance. For this purpose the teacher has to make use of descriptive and inferential analysis. Descriptive analysis aids in describing an entire set of scores with a single score. For example, mean (M) and standard deviation (SD) are two frequently used methods of descriptive analysis. Means provide representative score for an entire set of scores while SD is an index of the variations of the test scores from the mean value. By comparing the values of SDs the teacher can compare the groups in terms of their homogeneity or heterogeneity. Thus statistical analysis and interpretation help in drawing some meaningful conclusions.

9.3.1 Steps Involved in Statistical Analysis and Interpretation

- *Collection of Data:* The data has to be collected using the primary and secondary methods depending on the purpose for which data is collected. In order to use statistical methods of analysis, data collected should be tabulated in the numerical form.
- *Classification of Data:* Classification is the process of arranging data according to certain points of similarities or dissimilarities. Data can be classified on the basis of certain attributes or according to class intervals. Classification according to class intervals leads to formation of frequency tables.

- *Organisation and Presentation:* The data should be organised and presented in a concise form. The presentation of data in the form of class intervals and frequencies is known as a *frequency table.* The presentation of data in the form of a frequency table is very suitable for further statistical analysis.
- *Selecting Appropriate Statistical Technique for Analysis:* An appropriate method of analysis should be selected. The appropriateness of technique depends on the purpose for which data is collected and the level of accuracy required or expected.
- *Applying Selected Method of Analysis:* The data should be subjected to statistical treatment by applying the methods selected for analysis. At this step the computations are done and results are obtained.
- *Interpretation of Results:* This is the most important and crucial step. At this stage the results are interpreted and conclusions are drawn. Many vital decisions are taken based on the conclusions drawn by interpreting the results. Therefore, extreme care should be taken to draw right conclusions by meaningful interpretation of the statistical results.

Since the scope of the present topic does not include a detailed presentation of a statistical methods and analysis, only a brief account of descriptive analysis is presented.

9.4 Types of Descriptive Analysis

Measures of Central Tendency

When we observe any data, we find that most of the items are gathering together or clustering around a particular point. This point is called *point of central tendency.* This method of central tendency or averages give us a point which is most representative of the entire data. There are two main objects for the study of measures of central tendency.

- To get one single value that represents the entire data.
- To facilitate comparison.

There are three measures of central tendency or averages in common use. They are:

i) Arithmetic Mean
ii) Median and
iii) Mode

9.4.1 Arithmetic Mean

The most commonly used and familiar index of central tendency for a set of raw data or a distribution is the mean. The mean is simple arithmetic average. The arithmetic mean of a set of values is their sum divided by their number. The elementary procedure for obtaining a mean is to add all the values and divide the total by the total number of values, N. Consider the following scores or measurements, 8, 14, 23, 0, 12, 5. The sum of these scores is 72. The arithmetic mean is, therefore, 72 divided by 6 i.e. 12 (72/6 = 12). If N measurements are represented by the symbols X_1, X_2, X_3 .....X_n the arithmetic mean is

$$M \text{ or } (\overline{X}) = \frac{\text{Sum of the item}}{\text{Number of items}}$$

$$= \frac{X_1 + X_2 + X_3 ..... X_n}{N} = \frac{\Sigma X}{N}$$

ΣX. describes the summing of N measurements. Generally the arithmetic mean is written as $\overline{X} = \frac{\Sigma X}{N}$

Calculation of Mean From Frequency Distribution

Table-9.11: Calculation of Mean from the Grouped Data

Scores X	*Frequency* f	fx	*Computation formula* $\overline{X} = \frac{\Sigma fx}{N}$
16	2	32	
15	3	45	$\overline{X} = \frac{266}{20}$
14	4	56	
13	5	65	$= 13.3$
12	3	36	
11	2	22	
10	1	10	
	$\Sigma f = 20$	$\Sigma fx = 266$	

In the above frequency distribution each X is multiplied by the corresponding frequency and the product fx is obtained. The sum of the

product fx is taken and is denoted as Σfx. When we divide Σfx by Σf or N, the sum of the frequencies, we get the value of the arithmetic mean.

$$\text{Thus } = \frac{\Sigma fx}{N}$$

Calculation of Mean from Frequency Distribution

There are two methods of computing mean from frequency table.

- Arithmetic Mean by Long Method
- Arithmetic Mean by Short Method

Arithmetic Mean by Long Method

Table-9.12:Calculation of Mean from Frequency Distribution (Long Method)

(1) Class Interval	*(2) Frequency f*	*(3) Midpoint X*	*(4) fX*	*Computation formula*
80-90	1	85	85	
70-80	2	75	150	
60-70	6	65	390	$\overline{X} = \frac{\Sigma fx}{N}$
50-60	10	55	550	$= \frac{1890}{40}$
40-50	8	45	360	$= 47.50$
30-40	6	35	210	
20-30	4	25	100	
10-20	3	15	45	
	N = 40		$\Sigma fX = 1890$	

Table 9.12. illustrates the computation of Mean by long method. It follows the following steps.

- Calculate the mid points of each class interval and denote by X (col. 3)
- Multiply each X and the corresponding frequency f.(col,4)
- Find the sum of the product fX and denote it by ΣfX
- Substitute the values of ΣfX and N in the formula $\overline{X} = \frac{\Sigma fx}{N}$

Arithmetic Mean by Short Method **(Assumed Mean Method)**

The long method of calculating mean as illustrated above, very often involves the handling of large numbers and requires tedious calculations. Hence to overcome these difficulties the "Assumed Mean Method" or simply the 'Short Method' is devised. The same is illustrated below.

Table-9.13: Calculation of Mean from a Frequency Distribution by Short Method or Assumed Mean Method

(1) Class-interval	(2) f	(3) Midpoint x	(4) $d=\frac{X-AM}{i}$	(5) fd
80 – 90	1	85	3	3
70 – 80	2	75	2	4
60 – 70	6	65	1	6
50 – 60	10	55	0	0
40 – 50	8	45	-1	-8
30 – 40	6	35	-2	-12
20 – 30	4	25	-3	-12
10 – 20	3	15	-4	-12
	N = 40			$\Sigma fd = 31$

Formula $\overline{X} = AM + \left(\frac{\Sigma fd}{N}\right)i$ where AM is the assumed mean.

$$d\text{ (deviation)} = \frac{X - AM}{i}$$

X = midpoint, AM = Assumed Mean

i = class interval size

fd – product of the frequency and the corresponding deviation.

$$\overline{X} = AM + \left(\frac{\Sigma fd}{N}\right)i \qquad \overline{X} = 55 + \left(\frac{-31}{40}\right)10$$

$$= 55 - \left(\frac{31}{40}\right)\times 10 = 47.25$$

Steps in computation

- Find the midpoints (X) of the class intervals (col.3)
- Take one of the midpoints preferably the midpoint of an interval somewhere near the centre of the frequency distribution as assumed mean (A.M). In the present case AM = 55
- Find the deviations (d) of the assumed mean from the midpoints using the formula $d = \frac{X - AM}{i}$ (col.4). However, 'd' values can be assigned almost mechanically. Starting with d = 0 for the class interval having the AM, go up assigning 'd' values of +1, +2, +3 etc. till the upper most class interval is reached. Once again starting at 'd' = 0, go down assigning d values of –1,–2,–3,.. etc. till the lowest class interval is reached. This is possible because all class intervals are of uniform size.
- Find the product of the frequency (*f*) and the corresponding deviation (d) (col.5)
- Find Σfd by adding up the product *fd*
- Substitute the value of AM, Σfd, N and i in the formula.

$$\overline{X} = AM + \left(\frac{\Sigma fd}{N}\right) i$$

Merits of Arithmetic Mean

- It is easy to understand and simple to compute
- Its value is based on each and every item of the data with the result that a change in any item would mean a change in the average itself
- It is most commonly used in further statistical computations.
- Arrangement of data is not required for computing mean.

Demerits

- It is unduly affected by the extreme items, i.e.: very small and very large values pull up or pull down the values of the average.
- In certain cases mean may give absurd results. For example, if we are finding out the average size of a family in a certain district it may come out to be 6.7 or 5.3. It is absurd because persons cannot be in fractions.
- Its value cannot be determined graphically

- Mean cannot be calculated for open-end class intervals like below 50, 100 and above.etc.

9.4.2 Median (Md)

The median (Md) may be defined as that value which divides a distribution into two parts such that an exactly equal number of scores fall above and below the point. In other words 50 per cent of the scores will be above the median and the remaining 50 per cent below it.

Computation of Median

Ungrouped Data

a) when there is an odd number of items

Computing the value of the median involves the following procedure.

- First arrange the data in the ascending or descending order of magnitude
- Take the value of the middle item. *For example*, consider the series 7, 6, 9, 10, 4. (N = 5, an odd number)

Arranging the scores in an ascending order: 4, 6, (7), 9,10, we find that 7 is the middle item and therefore 7, is the median. There are two items on either side of 7

b) When there is an even number of items

When there is an even number of items the average of the middle two scores is taken as the median. The scores must be arranged in the ascending or descending order.

For example consider the following sores. 6, 9, 3, 4, 10, 5.

Arranging the scores in the ascending order. 3, 4, (5, 6) 9, 10

$$\text{Median} = \frac{\text{sum of the middle two scores}}{2}$$

$$= \frac{5+6}{2} = 5.5$$

A formula for finding the median of a series of ungrouped scores is.

$$\text{Median} = \text{the } \left(\frac{N+1}{2}\right)\text{th measure in order of size.}$$

Calculation of Median from a Frequency Distribution

In calculating the median from data grouped in the form of a frequency distribution, the fundamental logic of calculation remains the same as described in relation to ungrouped data. The method is illustrated in the table 15.14. given below.

Table-9.14: Median from Frequency Distribution

(1) Class – interval	(2) Frequency f	(3) Cumulative Frequency F	(4) Exact limits
80-89	1	40	79.5-89.5
70-79	2	39	69.5-79.5
60-69	6	37	59.5-69.5
50-59	10	31	49.5-59.5
40-49	8	21	39.5-49.5
30-39	6	13	29.5-39.5
20-29	4	7	19.5-29.5
10-19	3	3	9.5-19.5
	N = 40		

A formula for calculating the Md when the data have been classified into frequency distribution is

$$\text{Median} = l + \left(\frac{\frac{N}{2} - F}{fm} \right) i$$

Where, l = exact lower limit of the CI in which Median lies (or the CI in which the $\frac{N^{th}}{2}$ item lies)

F = Cumulative frequency up to the lower limit of the CI containing median

fm = Frequency of the CI containing median

i = Size of class interval

Here, $l = 39.5$, F = 13; fm = 8; i = 10.

$$\text{Median} = 39.5 + \left(\frac{\frac{40}{2} - 13}{8} \right) \times 10$$

$$= 39.5 + \left(\frac{20 - 13}{8} \right) \times 10$$

$$= 39.5 + \frac{7 \times 10}{8}$$

$$= 48.25$$

Steps in Computation

- Write down the exact limits of the class intervals
- Compute the cumulative frequency as shown in col.3.
- Determine $\frac{N}{2}$, one half of the number of cases, in this example $\frac{40}{2} = 20$
- Identify the CI in which the $\frac{N^{th}}{2}$ item lies. In the present example it is the CI (40-49) with exact limits 39.5-49.5.
- Find the value of l the exact lower limit of the median class
- Find the cumulative frequency F, up to the lower limit of the C.I. which contains the median. In this example F=13, which is the cumulative frequency up to the lower limit of C.I. (40-49).
- Read the frequency fm of the interval which contains the median.
- Substitute the values of l, F, fm, $\frac{N}{2}$ and i in the formula Median = $l + \left(\frac{\frac{N}{2} - F}{fm} \right) i$

Merits of Median

- Like mean, median is simple to understand.
- Median is not affected by extreme items.
- Median is specially useful in qualitative phenomena like honesty, intelligence, efficiency etc.
- Median is the most appropriate average for distribution with open end classes. (Example: Below 40, 40-60, 60-80... above 100)

Demerits

i) It is not always rigidly defined

ii) It is generally not used in further statistical work.

9.4.3 Mode

The mode is strictly defined as the point on the scale of measurement with maximum frequency in a distribution. It is a point of maximum concentration on a scale of values. Usually it is the item of the variable which occurs the largest number of times.

Mode from Ungrouped Data

Case (i)

For determining mode, we have to ascertain how many times the various items have repeated themselves. The item which has repeated the maximum number of items would be referred to as the Mode.

Scores: 100, 120, 120, 100, 124, 132, 120

Mode = 120, Since 120 occurs the largest numbers of times (f = 3) , it is the value of the mode.

Case (ii)

It two adjacent scores have the same frequency and the frequencies are the highest in the distribution, then the mode is the sum of the two scores divided by two.

Example:

Scores: 100, 120, 101, 109, 102, 103, 101, 104, 101, 102, 105, 102

Mode = $\frac{101+102}{2} = 101.5$, since the adjacent scores 101 and 102 have the largest but equal frequency of 3 each, the average of these values is the mode.

Case (iii)

Scores: 100, 101, 110, 111, 113, 101, 113, 115

Modes = 101 and 113. The non adjacent values of 101 and 113 have the largest but equal frequency of two each. Hence this set of scores has two modes 101, and 113. It can thus be called bimodal. Thus we can have more than one mode for a given distribution.

Case (iv)

When all values occur with the same frequency, calculation of mode is not possible. In such cases, mode is indeterminate.

Calculation of Mode in a Frequency Distribution

When scores are grouped in frequency distribution, the mode is the midpoint of the interval with the largest frequency. Consider the following frequency distribution in table 15.15.

Table-9.15: Mode from Frequency Distribution

Class interval	*Frequency*	
35-39	3	Mode is the midpoint of class
30-34	4	interval 15-19 which has the
25-29	6	highest frequency of 9.
20-24	7	Hence Mode $= \frac{15+19}{2} = 17$
15-19	9	
10-14	6	
5-9	5	

This is also called the *crude mode* or the empirical mode. The crude mode is approximately equal to the *true mode* and serves most of the practical purposes. A formula for approximation of the true mode from the symmetrical or not badly skewed distributions is Mode = 3 Median –2 Mean.

However, a more accurate value of mode can be computed for the data presented in a frequency table using the formula

$$\text{Mode} = l + \left(\frac{f_2}{f_1 + f_2} \right) i$$

where l is the actual lower limit of the Mode class (Mode class is the C.I having the highest frequency).

f_1 is the frequency of the class interval preceding the Mode class

f_2 is the frequency of the class interval succeeding the Mode class

i is the size of the C.I.

Note: CI_s are arranged in the ascending order. The values of f_2 and f_1 will be interchanged if CI_s are arranged in the descending order.

From table 15.16, the mode class is 15-19 as it has the highest frequency

$$\therefore l = 14.5; f_2 = 7; f_1 = 6; i = 5$$

$$\therefore \text{Mode} = l + \left(\frac{f_2}{f_1 + f_2}\right) i$$

$$= 14.5 + \left(\frac{7}{7+6}\right) 5$$

$$= 14.5 + 2.69$$

$$= 17.19$$

Merits of Mode

- It is simple to understand because it is the most typical value of the series
- Its value is not affected by the presence of extreme items
- Mode can be determined graphically
- Mode is useful in such cases where it is desired to find out the most popular value of the series.

Demerits

– It is a measure having very limited practical value.

– It does not lend itself readily to further algebraic manipulations.

– It is not used for further statistical measures.

Guidelines for the use of Various Measures of Central Tendency

The following general rules may be helpful to the student so as to decide when to use the various measures of central tendency.

Mean is useful

- when scores are symmetrically or nearly symmetrically distributed around a central point.
- when the situation warrants a measure of central tendency, having the greatest stability.
- when one wants to compute S.D. and other statistics which are based upon the mean.

Median is useful

- when the exact midpoint separating the distribution into two equal halves is wanted.
- when extreme scores are present in the distribution which would markedly affect the mean.
- when it is desired that certain scores should influence the central tendency, but all that is known about them is that they are above or below the median.

Mode is useful

- when a quick and approximate measure of central tendency is all that is wanted.
- when only the most typical value is required.

For example- the most typical size of the shirt or shoes worn by an average man

9.5 Measures of Dispersion or Variability

A measure of dispersion is designed to state the extent to which individual items differ from their arithmetic mean.

Objects of Measuring Variation

Measures of variation can be utilised for the following.

i) To test the reliability of an average

ii) To serve as a basis for the control of variability.

iii) To compare two or more groups with regard to their variability i.e., a high degree of variation would mean little uniformity or consistency whereas a low degree of variation would mean great uniformity or consistency.

iv) To facilitate the use of other statistical measures.

Methods of Studying Variation

The following are the measures of variability and each of these provides a numerical index of the variability of the scores.

i) Range
ii) Semi Inter Quartile Range or Quartile Deviation (Q)
iii) Mean Deviation or Average Deviation (M.D) or (A.D.)
iv) Standard Deviation (S.D.)

9.5.1 The Range

Range is the indicator of the variability that is easiest and most easily ascertained, but is also the most unreliable. It is defined as the difference between the highest and lowest scores.

$$R = H - L$$

As evident from the formula, range takes into account the extreme values of the scores only and ignores others. Hence it suffers from the following limitations.

Limitations

- It is unreliable when N is small or when there are gaps in the distribution.
- A change in the value of either of the extreme scores leads to a change in the value of the range.
- It does not consider the value of the scores between the extreme scores.
- Range cannot be calculated for open end class intervals.
- Further statistical analysis are difficult to make.

Uses

Range can be used in the following situations.

- When the data are too scattered to justify the computation of a more precise measure of variability.
- When a knowledge of total spread is all that is wanted.
- When a quick and crude estimate of variability is all what is desired.

9.5.1.1 The Semi-Inter-Quartile Range or Q

From the above discussion, there is every reason to believe that if the dispersion of extreme items is discarded, the limited range thus established might be more instructive. For this purpose there has been developed a

measure called inter-quartile range, the range which includes the middle fifty per cent of the distribution i.e. one quarter of the observations at the lower end and another quarter of the observations at the upper end are excluded in computing the inter-quartile range (see fig.15.5). In other words inter-quartile range represents the difference between the third quartile and the first quartile. In notation, Inter-Quartile range = Q_3-Q_1.

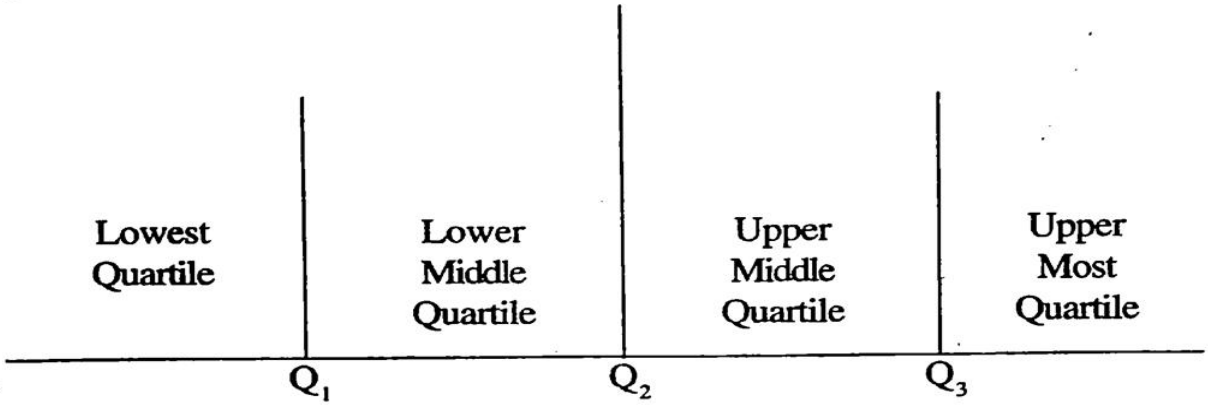

Fig.9.3: Diagrammatic Representation of the Quartiles

Half of the inter-quartile range or semi-inter-quartile range is called the *Quartile Deviation* (Q).

$$\text{Hence } Q = \frac{Q_3 - Q_1}{2}$$

Thus the Quartile Deviation is one half of the scale distance between the first and third quartiles in a frequency distribution. The first quartile (Q_1) on the score scale is the 25th percentile and the third quartile (Q_3) is the 75th percentile. Hence this first quartile is the point below which lies 25% of the scores and the third quartile is the point below which lies 75% of the scores.

Since Q measures the average distance of the quartile points from the median it is a good index of score density at the middle of the distribution. If the scores are packed closely together the quartiles will be near one another and Q will be small. If scores are widely scattered, the quartiles will be relatively far apart and Q will be large.

When the distribution is symmetric around the mean, or when it is normal Q marks exactly the 25% of the cases just below the median. The median thus lies just half way between the two quartiles Q_1 and Q_3.

To find out Q, it is essential to calculate values of Q_3 (or P_{75}) and Q_1 (or P_{25}). Their calculation follows the same procedure as the calculation of median as in table 15.14. The only difference is that $\frac{1}{4}$ of N is counted off

from the low end of the distribution to find Q_1 and that $\frac{3}{4}$ N is counted off to find Q_3.

The formulae are: $Q_1 = l_1 + \left(\frac{\frac{N}{4} - F_1}{f_1}\right) \times i$

$$Q_3 = l_3 + \left(\frac{\frac{3N}{4} - F_3}{f_3}\right) \times i$$

where l_1 = the exact lower limit of the interval in which the lower quartile falls.

i = the size of the interval

F_1 = the cumulative frequency up to the interval which contains the lower quartile.

f_1 = the frequency of the interval containing the lower quartile.

l_3 - The exact lower limit of the interval in which the upper quartile falls.

F_3 - Cumulative frequency up to the lower limit of the upper quartile class.

f_3 - The frequency of the upper quartile class.

Table-9.16: Calculation of Q_1, Q_3 and Q

Class-Interval	Frequency (f)	Cumulative frequency (F)	
45-49	2	50	
40-44	3	48	
35-39	2	45	
30-34	6	43	Q_3 lies in the C.I.
25-29	8	37	
20-24	8	29	
15-19	7	21	
10-14	5	14	Q_1 lies in the C.I
5-9	9	9	
	N = 50		

Calculation of Q_1

Here $l_1 = 9.5; F_1 = 9; f_1 = 5$

Substituting these values in the formula for Q_1, we have

$$Q1 = 9.5 + \left(\frac{\frac{50}{4} - 9}{5} \right) \times 5$$

$$= 9.5 + \left(\frac{12.5 - 9}{5} \right) \times 5 = 9.5 + 3.5 = 13.00$$

Calculation of Q_3

Here $l_3 = 29.5; F_3 = 37; f_3 = 6;$

$i = 5; N = 50$

Substituting these values in the formula for Q_3

$$Q_3 = 29.5 + \left(\frac{\frac{3 \times 50}{4} - 37}{6} \right) \times 5$$

$$= 29.5 + \left(\frac{37.5 - 37}{6} \right) \times 5$$

$$= 29.5 + 0.42 = 29.92.$$

Calculation of Q

Substituting the values of Q_1 and Q_3 in the formula $Q = \frac{Q_3 - Q_1}{2}$, we get

$$Q = \frac{29.92 - 13.00}{2} = \frac{16.92}{2} = 8.46.$$

Use the Q when

- the median is the measure of central tendency.
- there are scattered or extreme scores which would influence the SD disproportionately.

- the concentration around the median – the middle 50% of the cases – is of primary interest.

Merits of Q

- Superior to range as a rough measure of dispersion.
- Specially useful in case of open end classes.
- Not affected by the presence of extreme values.

Limitations

- As the value does not depend upon every item of the series, it cannot be regarded as a good method of dispersion.
- Affected by sampling fluctuations.
- It gives only a position on the scale rather than the scatter.

9.5.2 Average Deviation (AD)

The average deviation (or Mean deviation) is the average distance between the mean and scores in the distribution. In other words average deviation is the mean of the deviation of all the separate scores in a series taken from their arithmetic mean (occasionally from Median or Mode.). The deviation is defined as the distance of the score from the mean of the distribution. Algebraically, deviation can be defined as $\chi = X - M$, where X = original score, and M = arithmetic mean.

The sum of the deviations (with algebraic signs) from the arithmetic mean is always zero. Hence their average will always be zero and thus, useless for measuring and describing dispersion. Hence in averaging the deviation to find AD. no account is taken of signs and all deviations, whether plus or minus are treated as positive. The formula for average deviation is:-

$$AD = \frac{\Sigma|\chi|}{N}$$ where Σ = sum of; $|\chi|$ = absolute value of deviation and N = Total number of scores or observations.

Calculation of Average Deviation from Ungrouped Data

Table-9.17 illustrates the calculations of Average Deviation from ungrouped data.

Table-9.17: Calculation of AD from Ungrouped Data

X	$\chi = X-M$	$\lvert\chi\rvert$	
6	-4	4	
8	-2	2	$AD = \frac{\Sigma\lvert\chi\rvert}{N}$
10	0	0	
12	2	2	$= \frac{12}{5} = 2.4$
14	4	4	
$M = \frac{50}{5} = 10$	$\Sigma\chi = 0$	$\Sigma\lvert\chi\rvert = 12$	

Calculation of A.D. from Grouped Data

The table-9.18. illustrates the method of calculation of the AD from grouped data.

Table-9.18: Calculation of AD from Grouped Data

(1) Class Interval	(2) f	(3) Mid point X	(4) $\lvert\chi\rvert=\lvert X-M\rvert$	(5) $\lvert fx\rvert$	
195-199	1	197	26.20	26.20	
190-194	2	192	21.20	42.40	Mean (M) = 170.80
185-189	4	187	16.20	64.80	
180-184	5	182	11.20	56.00	
175-179	8	177	6.20	49.60	$AD = \frac{\Sigma\lvert fx\rvert}{N}$
170-174	10	172	1.20	12.00	
165-169	6	167	3.80	22.80	
160-164	4	162	8.80	35.20	$= \frac{502}{50} = 10.04$
155-159	4	157	13.80	55.20	
150-154	2	152	18.80	37.60	
145-149	3	147	23.80	71.40	
140-144	1	142	28.80	28.80	
	N=50			$\Sigma\lvert fx\rvert$=502.00	

In column (4) are entered the deviations (x) of each midpoint (X) from the Mean (M). In this case M=170.80. The deviation (x) for each midpoint (X) from the Mean (170.80) is calculated using the formula |X–M| and entered as shown in column (4). Each x deviation in column (4) is now multiplied by the frequency to get the entries |*fx*| in column (5). The first |x| of 26.20 is multiplied by 1; the second |x| of 21.20 by 2 and so on The sum of the |*fx*| (column 5) is taken and divided by N to give the AD. The formula is

$$AD = \frac{\Sigma|fx|}{N}$$

Substituting for $\frac{\Sigma|fx|}{N}$ in the formula,

$$AD = \frac{502}{50} = 10.04$$

Note: Mean is always subtracted from the midpoints i.e.(X–M = x)

The A.D when measured on the scale, above and below the mean includes 57.5% of the cases. Hence A.D. is always somewhat larger than the Q which includes the middle 50% of the items.

Steps in computing A.D

- Compute arithmetic mean (M) using either the direct or short method.
- Calculate deviation χ using the formula $\chi = \times - M$
- Find the absolute values of χ ie $|\chi|$; (Col.4)
- Multiply $|\chi|$ with the corresponding frequency to get $|f\chi|$; (Col. 5)
- Find the sum of $|f\chi|$ ie $\Sigma|f\chi|$
- Substitute the value of $\Sigma|fx|$ in formula $AD = \frac{\Sigma|fx|}{N}$

Use the A.D.

- When it is desired to weigh all deviations from the mean according to their size.
- When extreme deviations would influence S.D .unduly.

9.5.4 Standard Deviation (SD or σ)

The standard deviation or S.D. is the most reliable and stable index of variability. This concept was introduced by Karl Pearson in 1893. Its significance lies in the fact the it is free from those defects which the earlier methods suffer and satisfies most of the properties of a good measure of dispersion.

Like A.D, the S.D. is also a kind of average of all the deviations about the mean in a sample, though it is not a simple arithmetic mean. In computing the AD, we disregard signs and treat all deviations as positive, whereas in finding SD we avoid the difficulty of signs by squaring the separate deviations. Again the squared deviations used in computing the SD are always taken from the mean, never from the median or mode. The convention symbol for the SD is the Greek letter sigma. Standard deviation has been termed so because it provides a standard unit for measuring distances of various score from their mean.

In verbal terms, standard deviation is the square root of the arithmetic mean of the squared deviations of measurement from their mean. It has been often called as Root- Mean Square Deviation.

Calculation of the SD from Ungrouped Scores

We may illustrate the calculation of the SD for an ungrouped set of data with the same 5 scores used to demonstrate the computation of AD. The mean of the 5 scores 6,8,10,12 and 14 is 10 and the deviation of the separate scores from the mean are –4, –2, 0, 2, and 4 respectively. When each of these 5 deviations is squared, we get 16, 4, 0, 4 and 16; the sum is 40 and N, of course is 5. The formula for σ when scores are ungrouped is

$$\sigma = \sqrt{\frac{\Sigma x^2}{N}}$$ and in our example

$$\sigma = \sqrt{\frac{40}{5}}$$ or 2.83.

The table.9.19 illustrates the method of computation

Table-9.19: Computation of SD from Ungrouped Score

X	$\chi = X - M$	χ^2	$\sigma = \sqrt{\frac{\Sigma x^2}{N}}$
6	-4	16	
8	-2	4	
10	0	0	Where $x = X - M$
12	2	4	
14	4	16	$\sigma = \sqrt{\frac{40}{5}} = 2.83$
M = 10		$\Sigma x^2 = 40$	

Calculation of SD from Grouped Data: (Short method)

You may recall the short method of calculating the mean. This method consists essentially in "guessing" or assuming a mean and later applying a correction to give the actual mean. The short method may also be used to advantage in calculating the SD. The short method of calculating σ is illustrated in table15.20.

The formula for σ by the short method is.

$$= i\sqrt{\frac{\sum fd^2}{N} - C^2}$$

in which 'fd²' is the sum of the squared deviations in units of class interval taken from the assumed mean. C^2 is the squared correction (ie: $c^2 = \left(\frac{\sum fd}{N}\right)^2$ in units of class interval and i is the size of the class interval

Table-9.20: Computation of S.D. from Grouped Data by the Short Method (Deviations taken from Assumed Mean)

Class Interval	*Midpoint X*	*f*	$d = \frac{X - AM}{i}$	*fd*	fd^2
(1)	(2)	(3)	(4)	(5)	(6)
45-49	47	2	5	10	50
40-44	42	3	4	12	48
35-39	37	2	3	6	18
30-34	32	6	2	12	24
25-29	27	8	1	8	8
20-24	22	8	0	0	0
15-19	17	7	-1	-7	+7
10-14	12	5	-2	-10	+20
5-9	7	9	-3	-27	+81
		N = 50		Σfd = 4	$\Sigma fd^2 = 256$

$$\sigma = i\sqrt{\frac{\sum fd^2}{N} - C^2}$$

where *i* stands for the size of class interval and C for correction.

In the given data $i = 5$; $\Sigma fd^2 = 256$; $N = 50$; and $C = \frac{\Sigma fd}{N}$;

$c^2 = \left(\frac{4}{50}\right)^2 = .6664$ substituting the values in the formula

$$\sigma = 5\sqrt{\frac{256}{50} - .0064}$$

$$= 11.31$$

Computational steps

- Find out the midpoints (X) of all the class intervals
- Take a midpoint as an assumed mean.(A M)

- Find deviation (d) using the formula $d = \frac{X - AM}{i}$
- Multiply columns (3) and (4) to obtain fd and sum up to obtain Σfd
- Multiply columns (4) and (5) to obtain fd^2 and sum up to obtain Σfd^2
- Find out the value of C^2 using the formula $C^2 = \left(\frac{\Sigma fd}{N}\right)^2$
- Substitute these values in the formula $\sigma = i\sqrt{\frac{\Sigma fd^2}{N} - C^2}$ and find the value.

Use S.D.

- When the measure of dispersion having the greatest stability is required.
- When extreme deviations should exercise a proportionally greater effect upon the variability.
- When coefficients of correlation and other statistics are subsequently to be computed.

GRAPHICAL REPRESENTATION OF DATA

Introduction

Graphical representation of data is for the purpose of easier interpretation. Facts and figures as such do not catch our attention unless they are presented in an interesting way. Graphical representation of data is the most commonly used interesting modes of presentation. The purpose of this unit is to make you familiar with this interesting mode of presentation.

Meaning of Graphic Representation of Data:

Graphic representation is another way of analysing numerical data. A graph is a sort of chart through which statistical data are represented in the form of lines or curves drawn across the coordinated points plotted on its surface.

Graphs enable us in studying the cause and effect relationship between two variables. Graphs help to measure the extent of change in one variable when another variable changes by a certain amount.

Graphs also enable us in studying both time series and frequency distribution as they give clear account and precise picture of problem. Graphs are also easy to understand and eye catching.

General Principles of Graphic Representation:

There are some algebraic principles which apply to all types of graphic representation of data. In a graph there are two lines called coordinate axes. One is vertical known as Y axis and the other is horizontal called X axis. These two lines are perpendicular to each other. Where these two lines intersect each other is called '0' or the Origin. On the X axis the distances right to the origin have positive value (see fig. 7.1) and distances left to the origin have negative value. On the Y axis distances above the origin have a positive value and below the origin have a negative value.

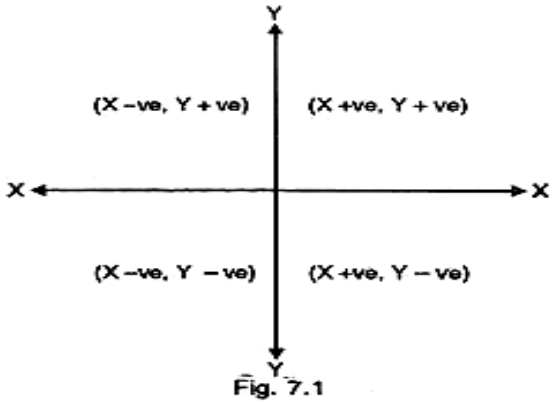

Fig. 7.1

Methods to Represent a Frequency Distribution:

Generally four methods are used to represent a frequency distribution graphically. These are Histogram, Smoothed frequency graph and Ogive or Cumulative frequency graph and pie diagram.

1. Histogram:

Histogram is a non-cumulative frequency graph, it is drawn on a natural scale in which the representative frequencies of the different class of values are represented through vertical rectangles drawn closed to each other. Measure of central tendency, mode can be easily determined with the help of this graph.

How to draw a Histogram:

Step—1: Represent the class intervals of the variables along the X axis and their frequencies along the Y-axis on natural scale.

Step—2: Start X axis with the lower limit of the lowest class interval. When the lower limit happens to be a distant score from the origin give a break in the X-axis n to indicate that the vertical axis has been moved in for convenience.

Step—3: Now draw rectangular bars in parallel to Y axis above each of the class intervals with class units as base: The areas of rectangles must be proportional to the frequencies of the corresponding classes.

Illustration No. 7.2

Plot the following data by a histogram.

c.i.	f
20—24	2
25—29	2
30—34	5
35—39	10
40—44	6
45—49	2
50—54	3

Solution: In this graph we shall take class intervals in the X axis and frequencies in the Y axis. Before plotting the graph we have to convert the class into their exact limits.

c.i.	f
19.5—24.5	2
24.5—29.5	2
29.5—34.5	5
34.5—39.5	10
39.5—44.5	6
44.5—49.5	2
49.5—54.5	3

Histogram plotted from the data.

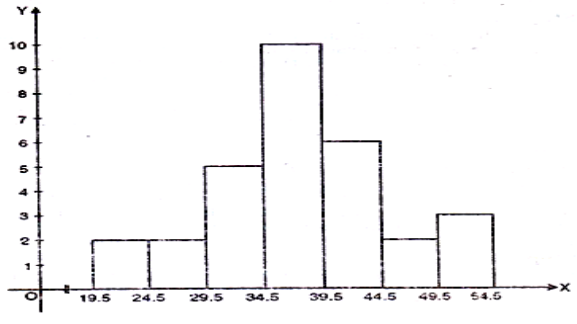

Fig. 7.2

Advantages of histogram:

1. It is easy to draw and simple to understand.
2. It helps us to understand the distribution easily and quickly.
3. It is more precise than the polygene.

Limitations of histogram:

1. It is not possible to plot more than one distribution on same axes as histogram.
2. Comparison of more than one frequency distribution on the same axes is not possible.
3. It is not possible to make it smooth.

Uses of histogram:

1. Represents the data in graphic form.
2. Provides the knowledge of how the scores in the group are distributed. Whether the scores are piled up at the lower or higher end of the distribution or are evenly and regularly distributed throughout the scale.
3. Frequency Polygon. The frequency polygon is a frequency graph which is drawn by joining the coordinating points of the mid-values of the class intervals and their corresponding frequencies.

Let us discuss how to draw a frequency polygon:

Step-1: Draw a horizontal line at the bottom of graph paper named 'OX' axis. Mark off the exact limits of the class intervals along this axis. It is better to start with c.i. of lowest value. When the lowest score in the distribution is a large number we cannot show it graphically if we start with the origin. Therefore put a break in the X axis () to indicate that the vertical axis has been moved in for convenience. Two additional points may be added to the two extreme ends.

Step-2: Draw a vertical line through the extreme end of the horizontal axis known as OY axis. Along this line mark off the units to represent the frequencies of the class intervals. The scale should be chosen in such a way that it will make the largest frequency (height) of the polygon approximately 75 percent of the width of the figure.

Step-3: Plot the points at a height proportional to the frequencies directly above the point on the horizontal axis representing the mid-point of each class interval.

Step-4: After plotting all the points on the graph join these points by a series of short straight lines to form the frequency polygon. In order to complete the figure two

additional intervals at the high end and low end of the distribution should be included. The frequency of these two intervals will be zero.

Illustration: No. 7.3:

Draw a frequency polygon from the following data:

Marks in Mathematics	40–45	45–49	50–54	55–59	60–64	65–69	70–74	75–79	80–84	85–89	90–95	95–99
No. of students	1	3	2	4	5	6	10	8	5	6	2	1

Solution:

In this graph we shall take the class intervals (marks in mathematics) in X axis, and frequencies (Number of students) in the Y axis. Before plotting the graph we have to convert the c.i(class interval). into their exact limits and extend one c.i. in each end with a frequency of O.

Class intervals with exact limits:

c.i.	f.
34.5–39.5	0
39.5–44.5	1
44.5–49.5	3
49.5–54.5	2
54.5–59.5	4
59.5–64.5	5
64.5–69.5	6
69.5–74.5	10
74.5–79.5	8
79.5–84.5	5
84.5–89.5	6
89.5–94.5	2
94.5–99.5	1
99.5–104.5	0

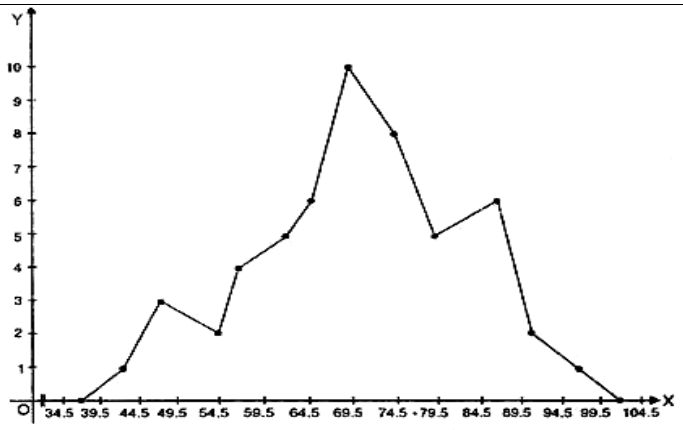

Fig. 7.3 Frequency polygon plotted from the data.

Advantages of frequency polygon:

1. It is easy to draw and simple to understand.
2. It is possible to plot two distributions at a time on same axes.
3. Comparison of two distributions can be made through frequency polygon.
4. It is possible to make it smooth.

Limitations of frequency polygon:

1. It is less precise.
2. It is not accurate in terms of area the frequency upon each interval.

Uses of frequency polygon:

1. When two or more distributions are to be compared the frequency polygon is used.
2. It represents the data in graphic form.
3. It provides knowledge of how the scores in one or more group are distributed. Whether the scores are piled up at the lower or higher end of the distribution or are evenly and regularly distributed throughout the scale.

2. Smoothed Frequency Polygon:

When the sample is very small and the frequency distribution is irregular the polygon is very jig-jag. In order to wipe out the irregularities and **"also get a better notion of how the figure might look if the data were more numerous, the frequency polygon may be smoothed."**

In this process to adjust the frequencies we take a series of 'moving' or 'running' averages. To get an adjusted or smoothed frequency we add the frequency of a class interval with the two adjacent intervals, just below and above the class interval. Then the sum is divided by 3. When these adjusted frequencies are plotted against the class intervals on a graph we get a smoothed frequency polygon.

Illustration 7.4:

Draw a smoothed frequency polygon, of the data given in the illustration No. 7.3:

Solution:

Here we have to first convert the class intervals into their exact limits. Then we have to determine the adjusted or smoothed frequencies.

c.i. (with exact limit)	f	Smoothed frequency
34.5—39.5	0	0 + 0 + 1 ÷ 3 = .33
39.5—44.5	1	0 + 1 + 3 ÷ 3 = 1.33
44.5—49.5	3	1 + 3 + 2 ÷ 3 = 2.00
49.5—54.5	2	3 + 2 + 4 ÷ 3 = 3.00
54.5—59.5	4	2 + 4 + 5 ÷ 3 = 3.67
59.5—64.5	5	4 + 5 + 6 ÷ 3 = 5.00
64.5—69.5	6	5 + 6 +10 ÷ 3 = 7.00
69.5—74.5	10	6 +10 + 8 ÷ 3 = 8.00
74.5—79.5	8	10 + 8 + 5 ÷ 3 = 7.67
79.5—84.5	5	8 + 5 + 6 ÷ 3 = 6.33
84.5—89.5	6	5 + 6 + 2 ÷ 3 = 4.33
89.5—94.5	2	6 + 2 + 1 ÷ 3 = 3.00
94.5—99.5	1	2 + 1 + 0 ÷ 3 = 1.00
99.5—104.5	0	1 + 0 + 0 ÷ 3 = .33

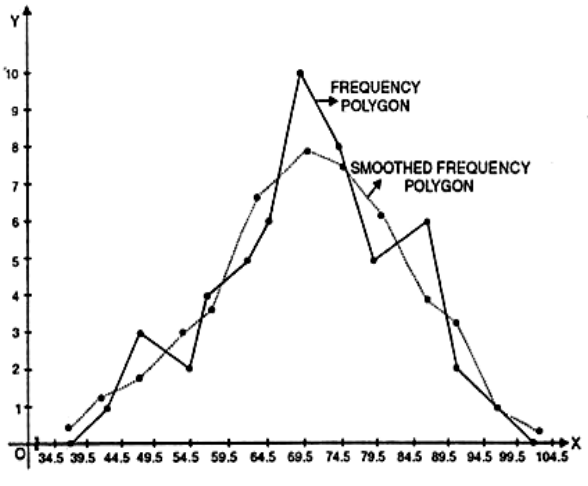

Fig. 7.4. Smoothed frequency polygon plotted from the data.

3. Ogive or Cumulative Frequency Polygon:

Ogive is a cumulative frequency graphs drawn on natural scale to determine the values of certain factors like median, Quartile, Percentile etc. In these graphs the exact limits of the class intervals are shown along the X-axis and the cumulative frequencies are shown along the Y-axis. Below are given the steps to draw an ogive.

- **Step—1:** Get the cumulative frequency by adding the frequencies cumulatively, from the lower end (to get a less than ogive) or from the upper end (to get a more than ogive).
- **Step—2:** Mark off the class intervals in the X-axis.
- **Step—3:** Represent the cumulative frequencies along the Y-axis beginning with zero at the base.
- **Step—4:** Put dots at each of the coordinating points of the upper limit and the corresponding frequencies.
- **Step—5:** Join all the dots with a line drawing smoothly. This will result in curve called ogive.

Illustration No. 7.5:
Draw an ogive from the data given below:

Marks in History	0—9	10—19	20—29	30—39	40—49	50—59	60—69	70—79	80—89	90—99
No. of Students	3	5	9	12	18	17	10	3	2	1

Solution:

To plot this graph first we have to convert, the class intervals into their exact limits. Then we have to calculate the cumulative frequencies of the distribution. Now we have to plot the cumulative frequencies in respect to their corresponding class-intervals. **Ogive plotted from the data given above:**

c.i.	f	c.f. (cumulative frequencies)
0—9.5	3	3
9.5—19.5	5	8
19.5—29.5	9	17
29.5—39.5	12	29
39.5—49.5	18	47
49.5—59.5	17	64
59.5—69.5	10	74
69.5—79.5	3	77
79.5—89.5	2	79
89.5—99.5	1	80

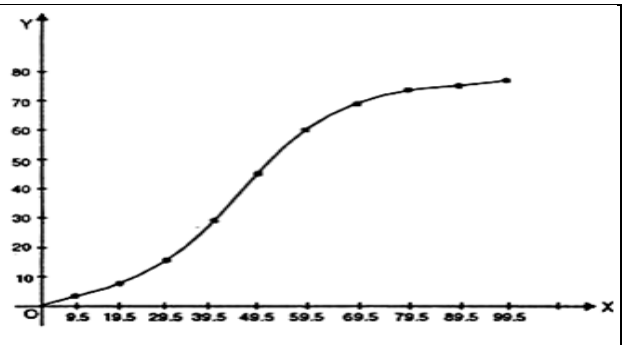

Fig. 7.5.

Uses of Ogive:

1. Ogive is useful to determine the number of students below and above a particular score.
2. When the median as a measure of central tendency is wanted.
3. When the quartiles, deciles and percentiles are wanted.
4. By plotting the scores of two groups on a same scale we can compare both the groups.

4. The Pie Diagram:

Figure given below shows the distribution of elementary pupils by their academic achievement in a school. Of the total, 60% are high achievers, 25% middle achievers and 15% low achievers. The construction of this pie diagram is quite simple. There are 360 degree in the circle. Hence, 60% of 360′ or 216° are counted off as shown in the diagram; this sector represents the proportion of high achievers students.

Ninety degrees counted off for the middle achiever students (25%) and 54 degrees for low achiever students (15%). The pie-diagram is useful when one wishes to picture proportions of the total in a striking way. Numbers of degrees may be measured off **"by eye"** or more accurately with a protractor.

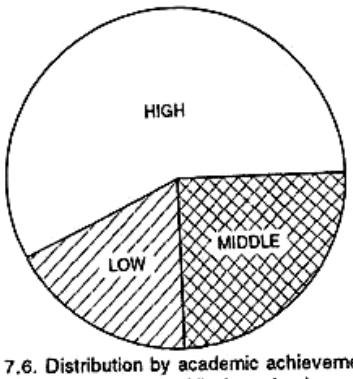

Fig. 7.6. Distribution by academic achievement of pupils in Class VI of a school.

Uses of Pie diagram:

1. Pie diagram is useful when one wants to picture proportions of the total in a striking way.
2. When a population is stratified and each strata is to be presented as a percentage at that time pie diagram is used.

1. RECREATIONAL ACTIVITIES IN MATHEMATICS

Mathematics is the mother of all sciences. Mathematics has its diverse significant values and aims or goals. So, it is regarded as one of the core part of the school curriculum across the world.

Recreational activities have a great role in mathematics learning and developing various skills in solving problem as well as developing creative and logical thinking. In one-way recreational mathematics is also a pure mathematics and is often difficult to distinguish pure mathematics from recreational mathematics. The pedagogic value of recreational mathematics is now widely recognized which in turn help the low achievers of mathematics and converting them into a lover of mathematics by removing fear-psychosis from their minds towards the subject. For a lover of mathematics, there is all beauty. One finds a huge treasure of pleasure after getting success in the solution of a Mathematics problem

Recreational mathematics has a vital role in making mathematics an interesting one. Experience shows that the basic principles of learning mathematics can be made easier through mathematical fun, activities and games. If mathematics can be turned into a game it can become child's play. Class room experiences indicate clearly that mathematical puzzles, riddles etc encourage an open minded attitude in youngsters and help them to develop their clear thinking. Shakuntala Devi become famous for her simple mathematical facts converting into game, puzzle etc.

DEFINITION:

Wikipedia defines recreational math:

'Recreational mathematics is mathematics carried out for recreation (entertainment) rather than as a strictly research and application-based professional activity. Although it is not necessarily limited to being an endeavour for amateurs, it often involves mathematical puzzles and games'

NEED OF RECREATIONAL ACTIVITIES IN MATHEMATICS

It is necessary to remove this indifference for mathematics from the minds of the students. They need to be motivated. Their interest in mathematics needs to be aroused and nurtured. Thus to remove the years old blame of the subject which creates fear-psychosis and hatred for the learner and to achieve higher aim of the subject which is responsible for developing a child's inner potentialities, the role of mathematics is vital, for that recreational activities plays a great role for all these.

Recreational activity is such a tool which not only help in developing different mathematical concepts but also help to retain those for a long time in the minds of the learner and thereby help in reproduce them as and when required. That is why curriculum for mathematics education must be ambitious and coherent with recreational activities. Also the text books of mathematics to be affordable to every child and at the same time enjoyable, which are also possible by incorporating Recreational activities.

IMPORTANTS/BENEFITS OF RECREATIONAL MATHEMATICS

The benefits of recreational math are many. The first and most obvious is that using math to solve a puzzle makes it more fun for students. With a goal of simply figuring out the puzzle, students will find themselves motivated to understand the math principles involved.

Some other benefits of recreational math include:

MOTIVATION

As mentioned above, motivating students—especially younger students—to learn and put math principles into play can prove very difficult. With a puzzle or game, students now have a direct reason to want to learn math.

SELF-GUIDING

Once they understand how to do a puzzle such as Sudoku, students do not need to be supervised. They learn as they go. On some websites, students can move through a variety of increasingly harder games as they learn more math skills.

IMPROVED SCORES

Games and puzzles improve student scores. That's true across all subjects, including math. Students who learn math through games have more success in remembering the lessons taught. They can also learn skills in solving grid puzzles that they will later use in solving math equations.

Some teachers have reported an overall improvement in math skills after learning how to solve recreational math puzzles.

Anything that can help young students enjoy learning about math is worthy of consideration by teachers. For those who learn how to get recreational math into the classroom, the benefits could prove substantial for their students.

Role of Recreational Activities in Mathematics Learning

Whatever good curriculum and textbook are prepared for mathematics education, its success and the learning of mathematics depends on the teacher and the way of their classroom transaction in a learner friendly environment. Here also recreational activities plays a vital role in developing and understanding a mathematical concept in an enjoyable way by relating it to the everyday life activities so that it can be permanently retained and use as and when necessary. Using Fun and Magic can make mathematics learning very exciting and interesting. Games provide opportunities for students to be actively involved in learning. They enable students to experience success and satisfaction, thereby build the enthusiasm and self confidence.

But these recreational activities are not only about fun and confidence building, they also, help students to –

¾ Understand Mathematical concepts

¾ Develop Mathematical skills

¾ Know Mathematical facts

¾ Lean the language of Mathematics

¾ Develop ability in mental thinking and reasoning

Many funs can be created in Mathematics learning in the classroom through various activities using TLM like match sticks, seeds, leaves, string, blocks, tangrams, geo-board, puzzles, songs, various games etc. These should be specially reflected in the text books at elementary stage otherwise teacher will not practice these ideas in the class room as teachers and learners are mostly dependent on the text book.

Role of Teacher Educator in different activities for mathematics learning

Teacher educators have one of the key-position to meet the challenges of teaching mathematics effectively. Yes they have responded rightly to meet that challenges in various ways -

• Some have developed powerful pedagogical approaches or teaching learning materials,

• Some have introduced innovations in their teaching,

• Some have worked with teachers,

• Some with students and

• Some others have taken up research to understand more deeply the teaching and learning of mathematics.

Following examples witnessed some such challenges through different recreational activities during long and short term training courses.

- Group forming games by applying mathematical operations creates very much excitement even among the trainees.
- Housie game is also a brain storming activity creates enthusiasm among the trainees .

Likewise many more activities are there. These activities help in drilling the known facts and prepared the trainees/students to grasp any new concept or understand the same properly.

TYPES OF RECREATIONAL MATHEMATICS

Recreational mathematics encompasses the following areas:

- Mathematical Puzzles
- Mathematical Games
- Mathematics Quiz
- Riddles
- Logic
- Magic
- Math and the arts
- Number theory
- Math humor

Mathematical games

Using games can make mathematics classes very enjoyable, exciting and interesting. Mathematical games provide opportunities for students to be actively involved in learning. Games allow students to experience success and satisfaction, thereby

building their enthusiasm and self-confidence. But mathematical games are not simply about fun and confidence building. Games help students to: • understand mathematical concepts • develop mathematical skills • know mathematical facts • learn the language and vocabulary of mathematics • develop ability in mental mathematics.

MATHEMATICS QUIZ

1. According to National policy of education 1986 has enforced that one of the main purpose of Teaching is to develop in children Mathematical attitude and mathematical…..discipline.

2. There is an axiom saying that "children of today are the artists of tomorrow". The Architect qualities can only be developed when the students are given various opportunities of learning and also should know the achievements and land marks of various branches of knowledge so the school should provide such activities to gain knowledge of all branches of mathematics or all branches of knowledge.

3. According in Kothari commission (1964-66). The destiny of nation is shaped in or through class room activities.

4. The National mathematics study committee report also stresses the need for teaching maths is to make children to think creatively to act creatively and to do creatively.

5. These things can be achieved by teaching mathematics more as a process than as a product and the very purpose of teaching mathematics is to provide to the children facts, concepts, making children to develop carefulness in making statements. To develop in them observation skill to develop in them a sense of inter relatedness, for this more direct teaching may not help the children so, that extended activities like mathematics Quiz (or) Brain storming sessions help the children to look at the situation in more mathematical way and to think relevantly about the situation.

6. Mathematics Quiz is a small test administered to know the students' knowledge or it is a short duration test used to know the student's knowledge and understanding in the field of Mathematics.

7. Mathematics Quiz is one of the activities with the help of which a mathematics teacher can motivate and encourage the children to become truthful, active, and alert and to arose in him/her critical thinking importantly frankness straight forwardness.

8. The mathematics quiz will go a long way in shaping and molding the personality of child towards mathematics.

9. Mathematics quiz is a means to bring a path in the mind a way of habit of reasoning or thinking.

II. Importance of Mathematics Quiz:

1. A Mathematics Quiz is a co-curricular activity it is a activity very much different from the usual class room activities. The class room activities are also called as controlled activities, the mathematics Quiz is an uncontrolled activity. It helps the children to develop the discipline on their own, towards mathematics.
2. Mathematics Quiz plays a vital role in seeking voluntary participation from the children.
3. Mathematics Quiz will be easy for those children who have the habit of wide reading.
4. Mathematics Quiz helps the children to gain perfect, authentic, accurate information.
5. Mathematics Quiz develops competitiveness among children.
6. It helps the children to enhance their knowledge.
7. It develops mathematical attitude in the children.
8. The children will cautions before making statements in Mathematics Quiz.

III. Objectives of Mathematics Quiz:

1. To develop critical thinking among children.
2. To make the children to learn mathematical attitude.
3. To develop the skill of answering precisely.
4. To make the children to think and answer quickly.
5. To consolidate and rapid revision of mathematical facts.
6. To develop a situation for children to understand and to answer immediately.
7. To exercise the practice of brief answering in the limited time.
8. To get equipped with useful and important facts.
9. To motivate the children to win prizes.
10. To faster creative instincts among children.
11. To make children to get an awareness about the contribution of mathematics and latest discoveries.

Organising mathematics Quiz: Organising mathematics Quiz and Execution of the same is a complex and challenging task. It requires a skill on the part of Teachers (or) the Quiz maker. The

whole program must be well planned in advance. To conduct the Mathematics Quiz successfully the following points has to be kept in mind. They are:

1. To conduct the Quiz we have to select the participants or we can say the volunteer on whom Quiz has to be conducted selected is random and has a criteria.

2. Duration of the Quiz: (Time)

It must be about 90 mints, with in this time we have to have as many rounds as possible; it depends on the number of rounds and also types of rounds.

3. **Groups:** There must be only 3 or 4 groups, minimum number of students has to be selected, and the questions of one round should not be repeated in any round.

4. Instruction: Instruction has to be given to all teams well in advance before starting each round. What they have to do? How they have to do? When they will gain marks? When they lose marks? Etc.

5. Rounds in the Quiz: In Mathematics Quiz there will be usually six rounds. They will be as follows.

I Round: In this round the Questions are asked to test the knowledge of mathematics.

II Round: In this round the Questions are asked to test the understanding of mathematics.

III Round: To test their transfer of learning Mathematical concepts and principles.

IV Round: Visual Round, to know their visual ability.

V Round: Buzzer round, the rapidness (or) Quickness in answering is also counted.

VI Round: Motor skill Round means shows their quickness in assembling things.

The Quiz has to be conducted with a moderator under moderator there will be four teams, moderator pose questions on the teams. There will be a time keeper. There will be a scorer with which we can conduct the Quiz. The score taken by each round. After announcing the final scores, who has got more marks gets the prize. Buy this the mathematics Quiz will be concluded.

A magic square as a recreational activity

A magic square is an NxN matrix in which every row, column, and diagonal add up to the same number. Ever wonder how to construct a magic square?

In recreational mathematics, a magic square is a square grid (where is the number of cells on each side) filled with distinct positive integers in the range such that each cell contains a different integer and the sum of the integers in each row, column and

diagonal is equal.The sum is called the magic constant or magic sum of the magic square. A square grid with cells on each side is said to have order n.

In regard to magic sum, the problem of magic squares only requires the sum of each row, column and diagonal to be equal, it does not require the sum to be a particular value. Thus, although magic squares may contain negative integers, they are just variations by adding or multiplying a negative number to every positive integer in the original square.

Magic squares are also called normal magic squares, in the sense that there are non-normal magic squares[5] which integers are not restricted in . However, in some places, "magic squares" is used as a general term to cover both the normal and non-normal ones, especially when non-normal ones are under discussion. Moreover, the term "magic squares" is sometimes also used to refer to various types of word squares

MATHEMATICAL PUZZLES:

Mathematical puzzles require mathematics in order to solve them. They have specific rules, as do multiplayer games, but mathematical puzzles don't usually involve competition between two or more players. Instead, in order to solve such a puzzle, the solver must find a solution that satisfies the given conditions.

Logic puzzles are a common type of mathematical puzzle. Conway's Game of Life and fractals are also considered mathematical puzzles, even though the solver only interacts with them by providing a set of initial conditions. As they often include or require game-like features or thinking, mathematical puzzles are sometimes also called mathematical games.

Mathematical puzzles make up an integral part of recreational mathematics. They have specific rules as do multiplayer video games, but they do not usually involve competition between two or more players. Instead, to solve such a puzzle, the solver must find a solution that satisfies the given conditions. Mathematical puzzles require mathematics to solve them. Logic puzzles are a common type of mathematical puzzle.

- Conway's Game of Life and fractals, as two examples, may also be considered mathematical puzzles even though the solver interacts with them only at the beginning by providing a set of initial conditions. After these conditions are set, the rules of the puzzle determine all subsequent changes and moves. Many of the puzzles are well known because they were discussed by Martin Gardner in his "Mathematical Games" column in Scientific

American. Mathematical puzzles are sometimes used to motivate students in teaching elementary school math problem solving techniques.

LIST OF MATHEMATICAL PUZZLES:

The following categories are not disjoint.

- **NUMBERS, ARITHMETIC, AND ALGEBRA**
  - Cross-figures or Cross number Puzzle
  - Dyson numbers
  - Four fours
  - KenKen
  - Liquid Water Pouring Puzzles
  - Feynman Long Division Puzzles
  - Pirate loot problem
  - Verbal arithmetics
  - 24 Game
- **COMBINATORIAL**
  - Cryptograms
  - Fifteen Puzzle
  - Kakuro
  - Rubik's Cube and other sequential movement puzzles
  - Str8ts a number puzzle based on sequences
  - Sudoku
  - Sujiko
  - Think-a-Dot
  - Tower of Hanoi
  - Diamond 25
  - Bridges Game
- **ANALYTICAL OR DIFFERENTIA**
  - Ant on a rubber rope
- **PROBABILITY**
  - Monty Hall problem
- **TILING, PACKING, AND DISSECTION**
  - Bedlam cube
  - Conway puzzle

- Mutilated chessboard problem
- Packing problem
- Pentominoes tiling
- Slothouber–Graatsma puzzle
- Soma cube
- T puzzle
- Tangram

- **INVOLVES A BOARD**

  - Conway's Game of Life
  - Mutilated chessboard problem
  - Peg solitaire
  - Sudoku

- **CHESSBOARD TASKS**

  - Eight queens puzzle
  - Knight's Tour
  - No-three-in-line problem

- **TOPOLOGY, KNOTS, GRAPH THEORY**

  The fields of knot theory and topology, especially their non-intuitive conclusions, are often seen as a part of recreational mathematics.

  - Disentanglement puzzles
  - Seven Bridges of Königsberg
  - Water, gas, and electricity
  - Slitherlink

- **MECHANICAL**

  - Rubik's Cube
  - Think-a-Dot

- **0-PLAYER PUZZLES**

  - Conway's Game of Life
  - Flexagon
  - Polyominoes

OTHER ACTIVITIES:

Other curiosities and pastimes of non-trivial mathematical interest include:

- patterns in juggling

- the sometimes profound algorithmic and geometrical characteristics of origami
- patterns and process in creating string figures such as Cat's cradles, etc.

SOME EXAMPLES OF PUZZLES, RIDDLES & BRAIN TEASERS

LOGIC PUZZLES

One of the best **brain teasers and riddles** goes as follows:

There are three switches downstairs. Each corresponds to one of the three light bulbs in the attic. You can turn the switches on and off and leave them in any position. How would you identify which switch corresponds to which light bulb, if you are only allowed one trip upstairs?

ANS- Keep the first bulb switched on for a few minutes. It gets warm, right? So all you have to do then is … switch it off, switch another one on, walk into the room with bulbs, touch them and tell which one was switched on as the first one (the warm one) and the others can be easily identified 😃

LOGIC RIDDLES

Brothers and sisters, I have none but this man's father is my father's son. Who am I looking at?

ANS- This funny riddle often amuses both kids and adults. The answer is simple – the man is my son.

LOGIC QUESTIONS

You are shown two doors – one leading to hell and the second one to heaven and only the door guards know what is behind the doors. One guard is always lying and the other is always telling the truth (of course, you don't know who is lying). You can ask only one guard one question.

What question can get you to heaven?

EINSTEIN'S RIDDLE

A fellow encountered a bear in a wasteland. Both got frightened – fellow ran to the north, bear to the west. Suddenly the fellow stopped, aimed his gun to the south and shot the bear.

What color was the bear?

ANS -Albert Einstein allegedly made this riddle for his scholars.
A fellow encountered a bear in a wasteland. There was nobody else there. Both were frightened and ran away. Fellow to the north, bear to the west. Suddenly the fellow stopped, aimed his gun to the south and shot the bear. What color was the bear? If you don't know, this may help you: if the bear ran about 3.14 times faster than the fellow (still westwards), the fellow could have shot straight in front of him, however for the booty he would have to go to the south.

RIVER CROSSING PUZZLES

A farmer returns from the market, where he bought a she-goat, a cabbage and a wolf (what a crazy market  . On the way home he must cross a river. His boat is small and won't fit more than one of his purchases. He cannot leave the she-goat alone with the cabbage (because the she-goat would eat it), nor he can leave the she-goat alone with the wolf (because the she-goat would be eaten).

How can the farmer get everything on the other side in this **river crossing puzzle**?

Hide Answer

Take the she-goat to the other side. Go back, take cabbage, unload it on the other side where you load the she-goat, go back and unload it. Take the wolf to the other side where you unload it. Go back for the she-goat. That's it.

WATER & WEIGHING

If you had a 5-liter bowl and a 3-liter bowl, and an unlimited access to water, how would you measure exactly 4 liters?

ANS- Fill the 5-litre bowl and pour water to the 3-litre bowl, which you empty afterwards. From the 5-litre bowl pour the 2 remaining litres to the 3-litre bowl. Refill

the 5-litre bowl and fill in the 3-litre bowl (with 1 litre), so there stay the 4 required litres in the 5-litre bowl.

NUMBERS & SEQUENCES

The day before yesterday I was 25 and the next year I will be 28. This is true only one day in a year.
What day is my birthday?

ANS- He was born on December 31st and spoke about it on January 1st.

2. PROBLEMS OF LEARNING MATHEMATICS

PROBLEMS OF LEARNING MATHEMATICS AT THE SCHOOL STAGE

Any analysis of mathematics education in our schools will identify a range of issues as problematic. We structure our understanding of these issues around the following four problems which we deem to be the core areas of concern:

- A sense of fear and failure regarding mathematics among a majority of children,
- A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time,
- Crude methods of assessment that encourage perception of mathematics as mechanical computation, and
- Lack of teacher preparation and support in the teaching of mathematics.

Each of these can and need to be expanded on, since they concern the curricular framework in essential ways.

FEAR AND FAILURE

If any subject area of study evokes wide emotional comment, it is mathematics. While no one educated in Tamil would profess (or at the least, not without a sense of shame) ignorance of any Tirukkural, it is quite the social norm for anyone to proudly declare that (s)he never could learn mathematics.

While these may be adult attitudes, among children (who are compelled to pass mathematics examinations) there is often fear and anxiety. Mathematics anxiety and ‘math phobia’ are terms that are used in popular literature. In the Indian context, there is a special dimension to such anxiety. With the universalisation of elementary education made a national priority, and elementary education a legal right, at this

historic juncture, a serious attempt must be made to look into every aspect that alienates children in school and contributes towards their non-participation, eventually leading to their dropping out of the system. If any subject taught in school plays a significant role in alienating children and causing them to stop attending school, perhaps mathematics, which inspires so much dread, must take a big part of the blame.

Such fear is closely linked to a sense of failure. By Class III or IV, many children start seeing themselves as unable to cope with the demands made by mathematics. In high school, among children who fail only in one or two subjects in year-end examinations and hence are detained, the maximum numbers fail in mathematics. This statistic pursues us right through to Class X, which is when the Indian state issues a certificate of education to a student.

The largest numbers of Board Exam failures also happen in mathematics. There are many perceptive studies and analyses on what causes fear of mathematics in schools. Central among them is the cumulative nature of mathematics. If you struggle with decimals, then you will struggle with percentages; if you struggle with percentages, then you will struggle with algebra and other mathematics subjects as well.

The other principal reason is said to be the predominance of symbolic language. When symbols are manipulated without understanding, after a point, boredom and bewilderment dominate for many children, and dissociation develops. Failure in mathematics could be read through social indicators as well.

Structural problems in Indian education, reflecting structures of social discrimination, by way of class, caste and gender, contribute further to failure (and perceived failure) in mathematics education as well. Prevalent social attitudes which see girls as incapable of mathematics, or which, for centuries, have associated formal computational abilities with the upper castes deepen such failure by way of creating self-fulfilling expectations.

A special mention must be made of problems created by the language used in textbooks, especially at the elementary level. For a vast majority of Indian children, the language of mathematics learnt in school is far removed from their everyday speech, and especially forbidding. This becomes a major force of alienation in its own right.

DISAPPOINTING CURRICULUM

Any mathematics curriculum that emphasises procedure and knowledge of formulas over understanding is bound to enhance anxiety. The prevalent practice of school mathematics goes further: a silent majority give up early on, remaining content to fail in mathematics, or at best, to see it through, maintaining a minimal level of achievement. For these children, what the curriculum offers is a store of mathematical facts, borrowed temporarily while preparing for tests.

On the other hand, it is widely acknowledged that more than in any other content discipline, mathematics is the subject that also sees great motivation and talent even at an early age in a small number of children. These are children who take to quantisation and algebra easily and carry on with great facility. What the curriculum offers for such children is also intense disappointment.

By not offering conceptual depth, by not challenging them, the curriculum settles for minimal use of their motivation. Learning procedures may be easy for them, but their understanding and capacity for reasoning remain under exercised.

CRUDE ASSESSMENT

We talked of fear and failure. While what happens in class may alienate, it never evokes panic, as does the examination. Most of the problems cited above relate to the tyranny of procedure and memorization of formulas in school mathematics, and the central reason for the ascendancy of procedure is the nature of assessment and evaluation.

Tests are designed (only) for assessing a student's knowledge of procedure and memory of formulas and facts, and given the criticality of examination performance in school life, concept learning is replaced by procedural memory. Those children who cannot do such replacement successfully experience panic, and suffer failure.

While mathematics is the major ground for formal problem solving in school, it is also the only arena where children see little room for play in answering questions. Every question in mathematics is seen to have one unique answer, and either you know it or you don't.

In Language, Social Studies, or even in Science, you may try and demonstrate partial knowledge, but (as the students see it), there is no scope for doing so in mathematics. Obviously, such a perception is easily coupled to anxiety.

Amazingly, while there has been a great deal of research in mathematics education and some of it has led to changes in pedagogy and curriculum, the area that has seen little change in our schools over a hundred years or more is evaluation procedures in mathematics. It is not accidental that even a quarterly examination in Class VII is not very different in style from a Board examination in Class X, and the same pattern dominates even the end-of chapter exercises given in textbooks. It is always application of some piece of information given in the text to solve a specific problem that tests use of formalism. Such antiquated and crude methods of assessment have to be thoroughly overhauled if any basic change is to be brought about.

INADEQUATE TEACHER PREPARATION

More so than any other content discipline, mathematics education relies very heavily on the preparation that the teacher has, in her own understanding of mathematics, of the nature of mathematics, and in her bag of pedagogic techniques.

Textbook-centred pedagogy dulls the teacher's own mathematics activity. At two ends of the spectrum, mathematics teaching poses special problems. At the primary level, most teachers assume that they know all the mathematics needed, and in the absence of any specific pedagogic training, simply try and uncritically reproduce the techniques they experienced in their school days.

Often this ends up perpetuating problems across time and space. At the secondary and higher secondary level, some teachers face a different situation. The syllabi have considerably changed since their school days and in the absence of systematic and continuing education programmes for teachers, their fundamentals in many concept areas are not strong.

This encourages reliance on 'notes' available in the market, offering little breadth or depth for the students. While inadequate teacher preparation and support acts negatively on all of school mathematics, at the primary stage, its main consequence is this: mathematics pedagogy rarely resonates with the findings of children's psychology.

At the upper primary stage, when the language of abstractions is formalised in algebra, inadequate teacher preparation reflects as inability to link formal mathematics

with experiential learning. Later on, it reflects as incapacity to offer connections within mathematics or across subject areas to applications in the sciences, thus depriving students of important motivation and appreciation.

3. MATH & MUSIC

Math and music have always been considered closely connected in many ways. • It is widely believed that students who do well in music also excel in math. • Let's take a look at some of the basic components of music and see what math has to do with them.

RHYTHM IS TO MUSIC AS NUMBERS ARE TO MATH

- Rhythm measures time
  - ✓ Measure is the space between two bar lines on the staff that represents the division of time by which air and movement of music are regulated
  - ✓ When you play a few different notes together or even repeat the same note on an instrument, you create something called rhythm.

- Music is made up of sound.
  - ✓ Sound is made from repeating sound waves.
  - ✓ The musical pitch of each note has a corresponding frequency measured physically in hz (hertz) or cycles per second.
  - ✓ There are some important mathematical relationships between the notes played in music and the frequency of those notes.

Pythagoras

• The Greek octave had a mere five notes. • Pythagoras pointed out that each note was a fraction of a string. • Example: Lets say you had a string that played an A. The next note is 4/5 the length (or 5/4 the frequency) which is approximately a C. The rest of the octave has the fractions 3/4 (approximately D), 2/3 (approximately E), and 3/5 (approximately F), before you run into 1/2 which is the octave A

Ratios

• Pythagoras was excited by the idea that these ratios were made up of the numbers 1,2,3,4, and 5. • Why? • Pythagoras imagined a "music of the spheres" that was created by the universe.

• The 18th century music of J. S. Bach, has mathematical undertones, so does the 20th century music of Philip Glass.

Golden Ratio and Fibonacci

• It is believed that some composers wrote their music using the golden ratio and the Fibonacci numbers to assist them • Golden Ratio: 1.6180339887 • Fibonacci Numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21

How is Music related to Math?

Mathematics is involved in some way in every field of study known to mankind. In fact, it could be argued that mathematics is involved in some way in everything that exists everywhere, or even everything that is imagined to exist in any conceivable reality. Any possible or imagined situation that has any relationship whatsoever to space, time, or thought would also involve mathematics.

Music is a field of study that has an obvious relationship to mathematics. Music is, to many people, a nonverbal form of communication, that reaches past the human intellect directly into the soul. However, music is not really created by mankind, but only discovered, manipulated and reorganized by mankind. In reality, music is first and foremost a phenomena of nature, a result of the principles of physics and mathematics.

The math / music concepts explored include:

• Fractions / rhythm
• Sets and intersecting sets / keys and related keys
• The relationship between ratios, fractions, and decimals / frequency and **interval**
• Roots and powers / equal temperament tuning

Frequency and Wave Length:

As shown by the University of New South Wales and the Peabody Conservatory of Music, every pitch has a distinct frequency. This means that every note has a specific

wavelength. Mathematicians are able to graph these waves, creating visual, numeric representations of sound.

Duration:

A music metronome marking indicates how many beats occur per minute. For instance, if the metronome marking indicates sixty beats per minute, then every beat has the duration of one second. This means that all rests and notes in music involve mathematical divisions and multiplications of time duration. Specific note types indicate these divisions and multiplications. For example, the duration of a whole note lasts four times as long as a quarter note.

Counting

Musicians count beats as they perform as this ensures that they will give the correct duration of time to each rest and pitch. Counting beats also identifies precisely where the person is in the music. For example, if a conductor tells an orchestra to start "at the middle of the bar" this can be ambiguous, since some music uses multiple time signatures and complex, overlapping rhythms. If the conductor can explain with numbers where he is in the music based on counting procedures (e.g., "on beat four"), then he has eliminated this ambiguity.

Engineering

The University of New South Wales shows that musical acoustics involves how sound waves bounce off surfaces in the environment. Engineers thus examine the precise angles at which sound waves will hit architectural surfaces in order to get the clearest and most efficient sound production.

Instrument Production

In order to achieve a uniform pitch and establish playing standards, instrument and reed manufacturers have to make sure that instruments and reeds have the same basic dimensions. This means that they have to make specific calculations to keep instruments and reeds the same shape and size. For example, oboe players want the length of their reeds to be about 72 mm, since reed length affects pitch.

Theory and Harmony

Since the time of Pythagoras, music developed as a relationship between ratios of frequencies. For example, doubling the frequency of any pitch will give a pitch exactly one octave higher than the original pitch. Human ears hear these ratios as harmony. Over thousands of years, changes in the treatment of what ratios make up a scale have changed what sounds "in tune." If it weren't for these changes, musicians wouldn't be able to transpose well or have scales that start on different pitches sound the same. Musicians also wouldn't have the modern 12-tone row, where mathematical addition and subtraction is the basis of every scale degree, since treatment of pitch ratios is what determines how many notes are in a scale. Math thus is the foundation for much of music theory and harmony.

Mathematics on Music

Notation is a method of writing down music so that it can be performed vocally and instrumentally. It is a kind of code that allows other people to interpret music. These symbols tells us three basic things about music; how long or short is the sound (duration), how high or low is the sound (pitch), and how should the music be played (expression).

The words we say and the notes we sing have specific duration. Duration refers to how long tone and silence last. Some sound duration are short while others are long. Duration is determined by the kind of notes and rests used. In relation to mathematics, fractions and whole numbers can tell how this duration works.

Time Signatures and Rhythm

The most fundamental use of mathematics in Western music is its application in rhythm. Nearly all Western music operates within time signatures, or pulses that are grouped into sections. Most commonly, these sections (called "measures") come in groups of two, three or four, but can come in groups of five or seven or complex sub-groupings of smaller numbers. The choice of time signature determines the feel of the pulse and rhythmical lilt of the music.

Within the measures, the timing of the notes is measured by notation that dictates rhythm in terms of symbols that denote, essentially, fractions. When a musician reads rhythms, he knows how long to hold each note and rest by calculating what fraction of the whole measure that note or rest is indicated to last.

Counting and Conducting

In multi-player ensembles, musicians must count and calculate the number of measures and beats during which they don't play in order to stay in time with the rest of the ensemble. In very large ensembles, a conductor helps keep track of the numbers of measures by waving his baton (stick) in a repetitive numerical pattern according to the time signature with each pattern representing one measure.

Intervals and Chords

Mathematics plays an important role in the analysis of melody and harmony in music. When music theorists search for and discern patterns in how pitch is used in music, the relationship of different pitches to one another is discussed in terms of intervals and scale degree numbers. This allows theorists to keep track of trends and apply knowledge of patterns to new compositions.

Contemporary Music

Contemporary and very recent Western music uses mathematics in what is perhaps the most complex manner. Prominent art composers of the 20th century created new musical forms based in mathematics. The most famous of these forms is 12-tone music, a system of composition wherein each of the pitches of the Western scale are assigned numbers and a mathematical matrix is written to determine the order of the notes in a composition.

Unit 10: Identification of learning difficulties

Identification of Learning difficulties - Slow Learners in Mathematics – Meaning, Characteristics, Reasons for Slow Learning and learning difficulties: dyslexia, dysgraphia and dyscalculia - remedial measures.

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7.3 Causes for Slow Learning in Mathematics and Remedial Measures for the Backward

There are certain distinguishing characteristics which should be helpful in recognising the slow learners and low achievers in mathematics. National Council of Teachers of Mathematics, Washington (NCTM, 1967) defined low achievers as the child ranking the 30th percentile of the students' population in achievements in mathematics. Although there are many possible exceptions to any given pattern, the following might be safely listed as the characteristics of the slow learners.

7.3.1 Identification of Slow Learners

- Intelligence quotient below 90
- Has little drive
- Has short span of attention
- Has weak association memory
- Is a poor reader
- Has difficulty with abstractions
- Is not logical in thinking
- Lacks imaginations
- Is unable to detect his own errors
- Has little power to transfer training
- Is not creative in his thinking.

Once the slow learners are identified, the next task is to find out the causes for the mathematical backwardness and planning for appropriate remedial programmes.

7.3.2 Causes and Remedies

- *Physical Causes*

Backwardness may be due to some physical causes such as poor eyesight, hearing defect or any other physical ailment which do not allow the child to concentrate on studies. Remedy of all these causes lies with the physician or doctor but some sort of physical exercise may also help the child.

- *Lack of Interest*

The backwardness may be due to distaste for the subject which may be natural or acquired. This could lead to an unfavourable disposition towards

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learning mathematics. The distaste may be due to harsh and disinterested teachers, too much pressure by parents, faulty methods of teaching and learning etc.

The teacher has to develop in the learner a positive attitude towards the subject through patience and persistence. This is possible by using illustrative aids. The well graded exercises given to suit their abilities will also go a long way in improving their achievement in mathematics.

- *Mental Disability*

The mental disability may be inborn or caused by environmental factors. The child may have low IQ, mental conflict, inferiority complex, feeling of insecurity, anxiety, tension, fear, nervousness, maladjustment etc. Many of these mental disorders can be successfully tackled by a competent teacher with a conscious effort. Attitude of affection, sympathy and kindness can go a long way in this regard.

- *Lack of Mathematical Abilities*

Certain abilities like abstract reasoning, numerical ability, spatial ability, arithmetic reasoning, computational ability are prerequisites for success in mathematics. Slow learners may lack proficiency in one or more of these abilities. The teachers will have to test proficiency of the slow learners on these abilities and necessary training programmes should be implemented to improve the skills and abilities

- *Inappropriate Learning Experiences*

The inappropriate learning experiences provided in the mathematics class could lead to confusion resulting in misconception of the basic mathematical concepts. The teachers should plan the learning experiences which are simple and relevant for the slow learners to achieve the objectives and get the concepts clear and clarified. Remedial teaching has to be done in such cases where the slow learners need them. Remedial teaching has to be planned in such a way that the learning experiences provided would be different and would meet the special needs of the slow learners.

- *Irregular Study Habits*

Mathematics is a subject of logical sequence. Higher order concepts depend upon low order concepts. Rules and formulae are statements of relationship among these concepts. Therefore a student with irregular study habits will find it hard to understand and apply the mathematical laws and principles. The teacher should help such students to plan their study time

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properly and make them more regular and systematic. Drill and review also could help them in improving their performance.

- *Teacher's Indifference*

Many a time the mathematics teachers become impatient and show indifference to the slow learners who are slow in grasping mathematical ideas and concepts. Moreover slow learners may not be able to perform the mathematical tasks at the same rate as their counterparts in the class. This could result in frustration among the slow learners leading to low achievement.

A teacher could take more interest in the slow learners and understand their levels of learning. This will definitely boost up the self-confidence of the slow learners.

- *Ineffective Methods of Teaching*

The group methods of teaching are not very effective for the slow learners because their rate of learning, level of achievement and level of understanding are not the same as the other students in the class. The teacher has to give special attention to the needs of the slow learners. In the case of slow learners, methods of individualised instruction like Programmed Instruction, Computer Aided Instruction (CAI) and use of learning packages and modules could yield better result and facilitate effective learning. The teacher should also give individual attention to the slow learners in clarifying their doubts, in stimulating and in directing their thinking. This will enthuse in them a sense of well being, trust and confidence in the teacher.

- *Practice and Drill*

The slow learners need more concrete experiences for effective learning and more drill and practice for longer retention. The teacher has to provide them with such opportunities which would result in meaningful learning.

- *Lack of Facilities at Home*

When the child does not have adequate time and facilities for learning at home, it may lead to backwardness. The teacher can help such students by arranging supervised study, where the child can learn under the supervision of the teacher. In this connection the teacher can seek the help of the gifted children.

- *Family Background and Home Environment*

The uncongenial atmosphere at home, the negative attitude of the parents towards the subject, the pressure of the parents and so on could

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adversely influence the students' performance in mathematics. A teacher could deal with such children with patience and sympathy. The teacher has to change the attitude of the parents and students by interacting with them in a more meaningful manner.

- *Irregular School Attendance*

The irregularity in attendance causes a serious problem for mathematics learning as it creates a wide gap in the student's understanding of mathematical concepts. Mathematics being a sequential subject, the understanding of a concept depends upon an earlier concept. Once the link is lost, the learning becomes more complex and difficult.The teacher has to look into the causes of irregularity in attendance and help the students in the best possible ways.

- *Lack of Proficiency in Fundamental Skills and Lack of Understanding in Basic Mathematical Concepts*

Proficiency in mathematical operations and understanding in basic mathematical concepts are two factors that could contribute towards better achievement. The slow learners may lack in these two fundamental aspects. Adequate drill and practice can enhance the proficiency in fundamental skills while review and more relevant learning experiences can make the basic concepts clear for the students. Moreover the teacher has to insist on neatness in work, legibility in handwriting, accuracy in copying the numbers and placing the number digits, neatness and accuracy in drawing geometrical figures, etc. so as to improve their speed and accuracy in doing mathematical work.

7.3.3 Some Useful Classroom Techniques for Slow Learners

- Provide opportunities for the class to learn through several senses at a time such as seeing, hearing, manipulating dramatising and doing.
- Have daily routine, with surprises, as routine gives them a feeling of security.
- Frequent changes of activity are necessary because slow learners have a short span of interest. Provide variety within a period.
- Never put a child on the spot for an answer if he is dull.
- Give these pupils immediate satisfaction by checking their work as they do it.
- Make each daily lesson complete in itself so that the slow learners can learn it easily.

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- Never penalise a slow child by forcing him to work longer at mathematics than his brighter peers.
- Always prepare pupils for verbal problems. One or two thought problems each day are more effective than a long test at one time.
- Always make directions clear by writing them on the blackboard.
- Do not try to force the slow learners to do mathematics when they are not really interested.
- Try to think of new ways to review concepts.
- Break content into small repetitive steps and give easy exercises for immediate reinforcement.
- When a question is asked, break the questions into a number of simpler questions.
- Do not insist on verbal definitions and statement of rules if there are evidences that the child has understood the idea.
- Always introduce a new relationship with the simplest arithmetic or algebra possible so that the pupil can concentrate on concept itself and not get frustrated by tedious computation.
- If there are several approaches to a new concept, use one per lesson to avoid confusion.

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LEARNING DIFFICULTIES IN MATHEMATICS

Mathematics disabilities can arise at nearly any stage of a child's scholastic development. While very little is known about the neurobiological or environmental causes of these problems, many experts attribute them to deficits in one or more of five different skill types. These deficits can independently of one another or can occur in combination. All can impact a child's ability to progress in mathematics.

1. Incomplete Mastery of Number Facts.
2. Computational Weakness.
3. Difficulty Transferring Knowledge.
4. Making Connections.
5. Incomplete Understanding of the Language of Mathematics.
6. Difficulty Comprehending the Visual Spatial Aspects and Perceptual Difficulties.

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DYSCALCULIA

Dyscalculia is a specific learning disability in mathematics. Kids with dyscalculia may have difficulty understanding number related concepts or using symbols or functions needed for success in mathematics.

Dyscalculia is a common learning issue that impacts kind's ability to do mathematics. It doesn't just affect them at school, however. The challenges can also create difficulties in daily life. The good news is there are various supports and strategies that can help kids gain the skills they need. The more you know about your child's challenges, the better able you will be to get her the help she needs.

Kids with this learning issue have trouble with many aspects of mathematics. They often don't understand quantities or concepts like biggest vs. smallest. They may not understand that the numeral 5 is the same as the word five. (This skills are sometimes called number sense.).

Kids with dyscalculia also have trouble with the mechanics of doing mathematics, such as being able to recall mathematics facts. They may understand the logic behind mathematics, but not how or when to apply what they know to solve mathematics problems. They also often struggle with working memory. For example, they may have a hard time holding numbers in mind while doing mathematics problems with multiple steps.

Dyscalculia Signs and Symptoms

Dyscalculia can cause different types of mathematics difficulties. So symptoms may vary from child to child. Dyscalculia often looks different at different ages. It tends to become more apparent as kinds get older. But symptoms can appear as early as preschool.

Pre-school:

1. Has trouble learning to count and skips over numbers long after kinds the same age can remember numbers in the right order.
2. Struggles to recognize patterns, such as smallest to largest or tallest to shortest.
3. Has trouble recognizing number symbols (knowing that "7" means seven).
4. Doesn't seem to understand the meaning of counting. For example, when asked for five blocks, she just hands you an armful, rather than counting them out.

Grade School:

1. Has difficulty learning and recalling basic mathematics facts, such as 2+4=6.
2. Struggles to identify +, - and other signs, and to use them correctly.
3. May still use fingers to count instead of using more advanced strategies, like mental mathematics.
4. Struggle to understand words related to mathematics, such as greater than and less than.
5. Has trouble with visual-spatial representations of numbers, such as number lines.

Middle School:

1. Has difficulty understanding place value.
2. Has trouble writing numerals clearly or putting them in the correct column.

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3. Has trouble with fractions and with measuring things, like ingredients in a simple recipe.
4. Struggles to keep score in sports games.

High School:

1. Struggles to apply mathematics concepts to money, including estimating the total cost, making exact change and figuring out a tip.
2. Has a hard time grasping information shown on graphs or charts.
3. Has difficulty measuring things like ingredients in a simple recipe or liquids in a bottle.
4. Has trouble finding different approaches to the same mathematics problem.

Possible Causes of Dyscalculia

Researchers don't know exactly what causes dyscalculia. But they have identified certain factors that indicate it's related to how the brain is structured and functions.

Here are some of the possible Causes of dyscalculia:

1. Genes. 2. Brain development. 3. Environment. 4. Brain injury.

How can we help our child with Dyscalculia?

Our role is unique in giving the support and encouragement to our child which he/she needs.

Here are some of the things we can do:

1. Explore multisensory techniques for teaching mathematics we can use at home.
2. Discover software, apps and Chrome tools to help the child with mathematics.
3. Look into free online assistive technology tools for mathematics on the web.
4. Find board games to the child can play to build mathematics skills.
5. Learn ways to help build the child's self-esteem.
6. See what the child can say to self-advocate in grade school and middle school.
7. Get tips on how to be an advocate for our child at school.
8. Discover our child's strength.
9. Explore the collection of strategies to help with dyscalculia to get even more ideas. And be sure to visit parenting Coach, where we will find hundreds of age-specific, practical tips to work through social, emotional and behavioral challenges.
10. Connecting and trading tips with other parents in our online community.
11. Reach out to experts through our free Experts live events.
12. Learn about Parent Training and information centers a free local resource.

DYSGRAPHIA AND MATHEMATICS

Dysgraphia is a learning disorder that affects a person's ability to write. Dysgraphia is not identified as a disability or disorder in the Diagnostic and Statistical Manual of Mental Disorders, but it falls under the manual's specific learning disorder category as impairment in written expression.

This condition is generally caused by disease of or damage to the brain and it can reveal itself in different ways. Some common traits of people with dysgraphia include poor handwriting and

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spelling skills, difficulty putting thoughts into words on paper, and difficulty taking notes during lecture.

Dysgraphia doesn't limit itself to words, it also affects a students' ability to learn and apply mathematics skills. For instance, students with dysgraphia may,

- Have inconsistent spacing between numbers and symbols.
- Omit numbers, letters, and words in writing.
- Have difficulty copying numbers from the board.
- Avoid tasks involving drawing or writing.
- Have unusual posture or a strained pencil/pen grip when writing.
- Have illegible handwriting.
- Have difficulty organizing thoughts and working through steps in writing.

Accommodating Students with Dysgraphia

It's important to remember that students with dysgraphia have a learning disorder, they are not lazy. So, avoid using negative reinforcement and avoid placing students in situations that make them feel uncomfortable. For instance, don't force students with dysgraphia to solve problems on the board in front of the class. Understand that students with dysgraphia can become frustrated easily and let them know you are available as an educationally.

The aim of such accommodations in to provide students with dysgraphia what they need to be just as successful as their peers. Such accommodations should help level the playing field, not provide an academic advantage. Also keep in mind that, by law, accommodations listed on the student's individualized Education Plan or Section 504 plan must be provided.

Tips for Teaching Students with Dysgraphia

When teaching Mathematics to students with dysgraphia, consider some of these suggestions,

1. Model and encourage appropriate pencil grip and posture. Try different pencils, pens, and pencil grips with the student until the student finds a comfortable tool, and speak with the school occupational therapist for suggestions.
2. Have the student complete repetitive handwriting exercise starting with numbers, then equations, and finally mathematical word problems, if appropriate.
3. Teach information in small chunks followed immediately with opportunities to solve, apply and independently practice the mathematics concept. Provide opportunities for hands on practice.
4. Have students use interactive notebooks for mathematics, as they can help with organization. Inside the front cover of students' notebooks. Provide a quick reference sheet.

Allow the student to use a word processor or assistive technology device to reduce or eliminate handwriting tasks. An Alpha Smart, for example, is an assistive technology device that allows the student to type rather than write and the teacher can connect it to a computer to print the document.

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Dyslexia

Definition of Dyslexia

The World Federation of Neurology defines Dyslexia as —A disorder manifested by difficulties in learning to read, despite conventional instructions, adequate, intelligence and socio-cultural opportunities.

Signs of dyslexia

The following are some common signs of dyslexia:

- The dyslexic reader is more often a boy than a girl. The ratio is 4:1
- Dyslexic readers tend to be spatially disoriented, with pronounced left / right confusion. This affects the way they perceive words.
  - There is frequent reversal of letters [b-d, flim-film]; words [saw- was], and sometimes entire sentence.
  - Sounds are confused [empty- entry].
  - Concepts are reversed [floor for ceiling; cake for bread]
- There may be poor figure ground discrimination visual-motor difficulties. He may have untidy writing. The letters are poorly formed, irregular and uneven in formation.
- Reading rate is low
- Tends to make mistakes in loud reading, laborious reading, reads word by word mispronunciation occurs often.
- Poor reading comprehension
- Omission of letters, syllables, words or word endings, (calls for called / call, sed / said)
- Addition of sounds or words in sentences (ischool / school)
- Substitution of words / letter (home / house, ise cream / ice cream Mispronouncing words / letter (joo / zoo)
- Transposing of order of words in a sentence (I like to dance / dance like I to)

Types of Dyslexia

Visual Dyslexia: It is marked by:

- Reversal of whole word / syllable / letters (now / won, was / saw, b / d).
- Substitution (house / home)

- Addition of sounds (ischool / school)
- Omission of letters, syllables, words or word endings (sed / said). It arises due to child's short memory span and difficulty in interpreting and recalling visual images.

Auditory Dyslexia: It is characterized by the child

- having difficulty in processing and understanding what has been said to him.
- not being able to distinguish subtle differences in sound (bit/bet, pig/peg).
- having difficulty in filtering out the extraneous sound. It is quite likely that a child with auditory dyslexia becomes very tense or hyper-active in a noisy room.

Deep Dyslexia: It is a combination of the above two

Important components for successful reading

Phonological Awareness: Phonological Awareness or phoneme awareness means understanding that speech is made up of individual sounds - Phonemes – arranged in a particular sequence and the ability to manipulate the smallest units of sound (phonemes) that make up spoken words. There are forty-four different phonemes in the English language which are represented by one or more of the 26 letters in the English alphabet.

Phonemic Awareness

- It is not phonics. It is strictly oral; knowing how to manipulate the sounds in spoken words.
- It requires focusing on the sounds of speech, not the meaning.

Understanding that words can be segmented into syllables and speech sounds. e.g. car / pen / _cat' /c/ -/a/ -/t/

Alphabet knowledge: The ability to recognize and name the letters of the alphabet, symbols of a letter, sound of a letter appears to be the second most important

instructional factor in learning to read. It is highly co-related with later reading successes.

Basic Sight Words: A child can make little progress in reading without a basic sight vocabulary. This becomes especially important for the disabled reader. The Dolch basic sight vocabulary of 220 words contains about 65 % of all the words in the reading material of primary and secondary grades. If the child does not have these words in his sight vocabulary or cannot recognize them instantly, he cannot become a fluent reader. There are several lists of these common or basic sight words.

Spelling: Spelling is the forming of words from letters according to accepted usage; a sequence of letters composing a word; a statement of rules on how words are to be written. Spelling strengthens the skills needed for reading.

Fluency: The ability to read, speaks, or write isolated words, phrases and text accurately and quickly.

Substitution: The child substitutes words, which look and almost mean the same (home / house, jungle / forest). Sometimes there are bizarre insertions (bus / baking). The child focuses only on the initial part of the word and ignores the rest. The problem of substitution signifies that either the child has poor decoding ability or poor fluency skills. If the minor mistakes appear rarely, it is best to ignore the problem.

Repetition: The child re-reads words or phrases. This difficulty occurs because the child is either not able to make sense of the meaning or is anxious and nervous while reading or lack of confidence.